Final Exam : Next Monday . 3pm - 5pm at Ballentine Hall rom 310.

Office Hour : Sunday (12/12) . 8pm - 10pm
Next Monday (12/13) . 9am - 11am) in my Zoom meeting room.

HW 12 : due today at 11:59 pm.

Special HW : due Sunday (12/12) at 11:59 pm.

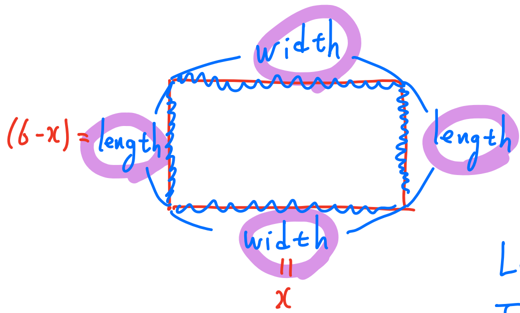
36. There is 12 feet of fencing available to enclose a rectangular region. For what widths will the fenced region have an area at least 8 square feet?

$(\text{length}) + (\text{width}) + (\text{length}) + (\text{width}) = 12$.

$2 \cdot (\text{length}) + 2 \cdot (\text{width}) = 12$.

$\downarrow \div 2$

$(\text{length}) + (\text{width}) = 6$.



Let x (feet) be the width.
Then the length is $(6-x)$ (feet).

$2 \text{ feet} \le (\text{width}) \le 4 \text{ feet}$

$(\text{area}) = \boxed{x \cdot (6-x) \ge 8}$

$6x - x^2 \ge 8$

$-(6x-x^2) \quad -(6x-x^2)$

$0 \ge 8 - (6x - x^2)$

$0 \ge 8 - 6x + x^2$

$x^2 - 6x + 8 \le 0$.

$(x-4)(x-2) \le 0$

$x-4=0 : x=4$ $\quad 0 \le 0$ true.

$x-2=0 : x=2$ $\quad 0 \le 0$ true.

$(x-2)(x-4)$	$+$		$-$		$+$
$x-4$	$-$		$-$	0	$+$
$x-2$	$-$	0	$+$		$+$
		2		4	

$(2, 4)$ add 2, 4 $\to [2, 4]$.

$[2, 4]$

$2 \le x \le 4$.

37. Four hundred people attended a movie. Adult tickets cost \$11 and children were admitted for \$7. How many children attended the movie if the total revenue from selling tickets was \$3,800.

	How many?	ticket price per a person.	total revenue.
Adult	$400 - \overset{150}{x} = 250$.	\$11.	$\$ \boxed{11 \cdot (400 - x)}$
Children	x	\$7.	$\$ \boxed{7 \cdot x}$

Let x be the number of children.

$(\text{Children}) + (\text{Adult}) = 400$.

$x \qquad 400 - x$!

$\Downarrow$

$11 \cdot (400 - x) + 7 \cdot x = 3{,}800$

$4400 - 11x + 7x = 3{,}800$

$4400 - 4x = 3{,}800$

$-4400 \qquad -4400$

$-4x = -600 \xrightarrow{\div(-4)} \boxed{x = 150}$

150 Children

38. Assuming that a constant interest rate of 4% ($r = 0.04$) per year is compounded continuously, determine how much would you have to invest today (P) in order to have ten thousand dollars ($A = 10{,}000$) after 10 years ($t = 10$).

- Continuously Compounded Interest Formula

$A = Pe^{rt}$, where P = principal

r = annual interest rate expressed as a decimal

t = number of years P is invested

A = amount after t years.

$$A = P \cdot e^{rt}$$

$$10{,}000 = P \cdot e^{0.04 \cdot 10}$$

$$10{,}000 = P \cdot e^{0.4}$$

$$\Big\downarrow \div e^{0.4}$$

$$\frac{10{,}000}{e^{0.4}} = P$$

$$\Rightarrow \boxed{\$\left(\frac{10{,}000}{e^{0.4}}\right)}$$

39. The price of the car after applying 30% tax is 39000 dollars. Find the price of the car before tax.

30% → tax rate is 0.3

39000 dollars → price after applying tax

(Tax) = (price before tax) × (tax rate)

(price after applying tax) = (price before tax) + (Tax)

Let $\$x$ be the price of the car before tax.

$(\text{Tax}) = (\$x) \times 0.3 = \$(0.3x)$

$\$39000 = \$x + \$(0.3x)$

$\$39000 = \$(1.3 \cdot x)$

$1.3 \cdot x = 39000$

$\frac{13}{10} x = 39000$

$\times \frac{10}{13}$

$\frac{\cancel{10}}{\cancel{13}} \cdot \frac{\cancel{13}}{\cancel{10}} \cdot x = \frac{10}{\cancel{13}_1} \cdot \cancel{39000}^{3000}$

$x = 30{,}000$

$\therefore \boxed{\$30{,}000}$

40. A clothing store holding a clearance sale advertises that all prices have been discounted 30%. If a pants is on sale for \$35, what was its $\boxed{\text{presale price?}}$

discount rate = 0.3

$$\underbrace{(\text{discount})}_{} = (\underbrace{\text{presale price}}_{\$x}) \times (\underbrace{\text{discount rate}}_{0.3}) \Rightarrow (\text{discount}) = x \cdot 0.3 = \$0.3 \cdot x$$

$$(\underbrace{\text{price on the market}}_{\$35}) = (\underbrace{\text{presale price}}_{\$x}) - (\underbrace{\text{discount}}_{\$0.3 \cdot x}) \Rightarrow 35 = x - 0.3x$$

Let \$x be the presale price.

$$35 = 0.7 \cdot x$$

$$35 = \frac{7}{10}x$$

$$\downarrow \times \frac{10}{7}$$

$$\cancel{35}^{5} \cdot \frac{10}{\cancel{7}_1} = \frac{\cancel{10}}{\cancel{7}} \cdot \frac{\cancel{7}}{\cancel{10}} \cdot x, \quad x = 50.$$

$$\Rightarrow \boxed{\$50}$$

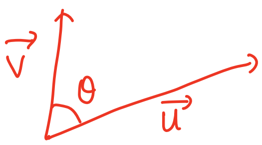
33. When $\mathbf{a} = \langle 4, 3\rangle$ and $\mathbf{b} = \langle -2, 3\rangle$, find $2\mathbf{a} + 3\mathbf{b}$

$m \cdot \langle a_1, a_2\rangle = \langle m \cdot a_1, m \cdot a_2\rangle$

$\langle a_1, a_2\rangle + \langle b_1, b_2\rangle = \langle a_1 + b_1, a_2 + b_2\rangle$

$$2 \cdot \mathbf{a} + 3 \cdot \mathbf{b} = 2 \cdot \langle 4, 3\rangle + 3 \cdot \langle -2, 3\rangle$$
$$= \langle 8, 6\rangle + \langle -6, 9\rangle$$
$$= \langle 8 + (-6), 6 + 9\rangle$$
$$= \boxed{\langle 2, 15\rangle}.$$

34. Find the angle between vectors $\vec{v} = \langle 0, 4\rangle$ and $\vec{u} = \langle \sqrt{2}, \sqrt{2}\rangle$



$$\cos\theta = \frac{\vec{v} \cdot \vec{u}}{\|\vec{v}\| \cdot \|\vec{u}\|}$$

$\langle a_1, b_1\rangle \cdot \langle a_2, b_2\rangle = a_1 \cdot a_2 + b_1 \cdot b_2$,

$\|\langle a_1, b_1\rangle\| = \sqrt{a_1^2 + b_1^2}$

$$\cos\theta = \frac{\langle 0, 4\rangle \cdot \langle \sqrt{2}, \sqrt{2}\rangle}{\|\langle 0, 4\rangle\| \cdot \|\langle \sqrt{2}, \sqrt{2}\rangle\|}$$
$$= \frac{0 \cdot \sqrt{2} + 4 \cdot \sqrt{2}}{\sqrt{0^2 + 4^2} \cdot \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}} = \frac{0 + 4\sqrt{2}}{\sqrt{16}\sqrt{4}}$$
$$= \frac{4\sqrt{2}}{4 \cdot 2} = \frac{\sqrt{2}}{2}.$$

$\cos\theta = \frac{\sqrt{2}}{2} \Rightarrow \boxed{\theta = \frac{\pi}{4} \text{ or } 45^\circ.}$

23. Find the specified term of the arithmetic sequence that has the two given terms.

a_3; $a_6 = 25$, $a_7 = 28$.

$a_1, a_2, a_3, a_4, \cdots \cdots$ ($+d$, $+d$)

$\boxed{a_n = a_1 + (n-1) \cdot d}$

$a_1 + 5d = a_6 = 25$

$a_1 + 6d = a_7 = 28$

$a_1 + 6d = 28 \longrightarrow a_1 + 6 \cdot 3 = 28$

$a_1 + 5d = 25$

$a_1 + 18 = 28$, $a_1 = 10$.

$(a_1 + 6d) - (a_1 + 5d) = 28 - 25$

$d = 3$.

$a_3 = a_1 + 2d$

$= 10 + 2 \cdot 3 = 10 + 6 = \boxed{16}.$

22. Find the sum (1) $\sum_{k=1}^{5} \underbrace{(7-2k)}_{} \text{''} a_k$

$= a_1 + a_2 + a_3 + a_4 + a_5$

$= (7-2\cdot1) + (7-2\cdot2) + (7-2\cdot3) + (7-2\cdot4) + (7-2\cdot5)$

$= (7-2) + (7-4) + (7-6) + (7-8) + (7-10)$

$= 5 + 3 + 1 + (-1) + (-3)$

$= \boxed{5}.$

(2) $\sum_{k=1}^{100} 3 \text{''} a_k$

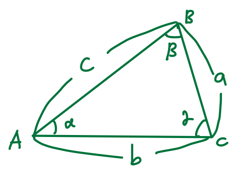
$= a_1 + a_2 + a_3 + \cdots + a_{100}$

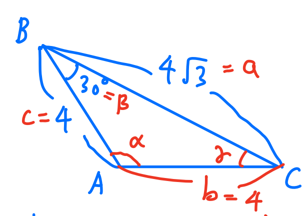
$= \overbrace{3 + 3 + 3 + \cdots + 3}^{100\ 3\text{'s}}$

$= 3 \cdot 100$

$= \boxed{300}.$

32. Solve $\triangle ABC$, given $a = 4\sqrt{3}$, $c = 4$, and $\beta = 30°$





The Law of Cosines (11/19)

For any triangle $\triangle ABC$, we have
$a^2 = b^2 + c^2 - 2bc \cos\alpha$
$b^2 = a^2 + c^2 - 2ac \cos\beta$
$c^2 = a^2 + b^2 - 2ab \cos\gamma$

By the law of Cosine,

$b^2 = (4\sqrt{3})^2 + 4^2 - 2\cdot4\sqrt{3}\cdot4\cdot\cos 30°$

$= 16\cdot3 + 16 - 2\cdot4\sqrt{3}\cdot4\cdot\frac{\sqrt{3}}{2}$

$= 48 + 16 - 48$

$= 16$.

$b^2 = 16 \Rightarrow \boxed{b = 4}$

Since $b = c = 4$, $\boxed{\gamma = \beta = 30°}$.

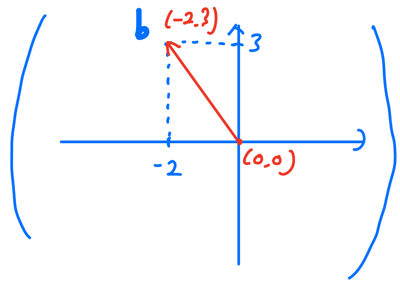
Then, $\boxed{\alpha = 180 - \beta - \gamma = 180 - 30 - 30 = 120°}$.

35. The magnitude and direction of a constant force are given by $\mathbf{a} = 2i + 7j$. Find the work done if the point of application of the force moves from the origin $O(0,0)$ to the point $P(-2, 3)$

Definition of Work

The work W done by a constant force $\mathbf{a}$ as its point of application moves along a vector $\mathbf{b}$ is $W = \mathbf{a} \cdot \mathbf{b}$.

In this problem, a constant force is $\mathbf{a} = 2i + 7j$, and the point move along a vector $\mathbf{b} = -2i + 3j$.



Hence, (the work done) $= \mathbf{a} \cdot \mathbf{b}$

$$
\begin{aligned}
&= (2i+7j) \cdot (-2i+3j) \\
&= \langle 2, 7 \rangle \cdot \langle -2, 3 \rangle \\
&= 2 \cdot (-2) + 7 \cdot 3 \\
&= -4 + 21 \\
&= \boxed{17}.
\end{aligned}
$$