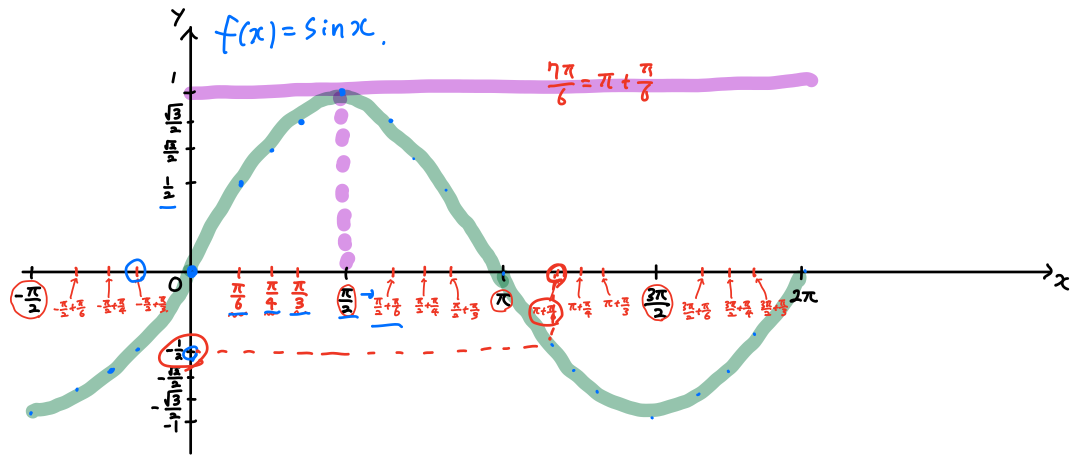
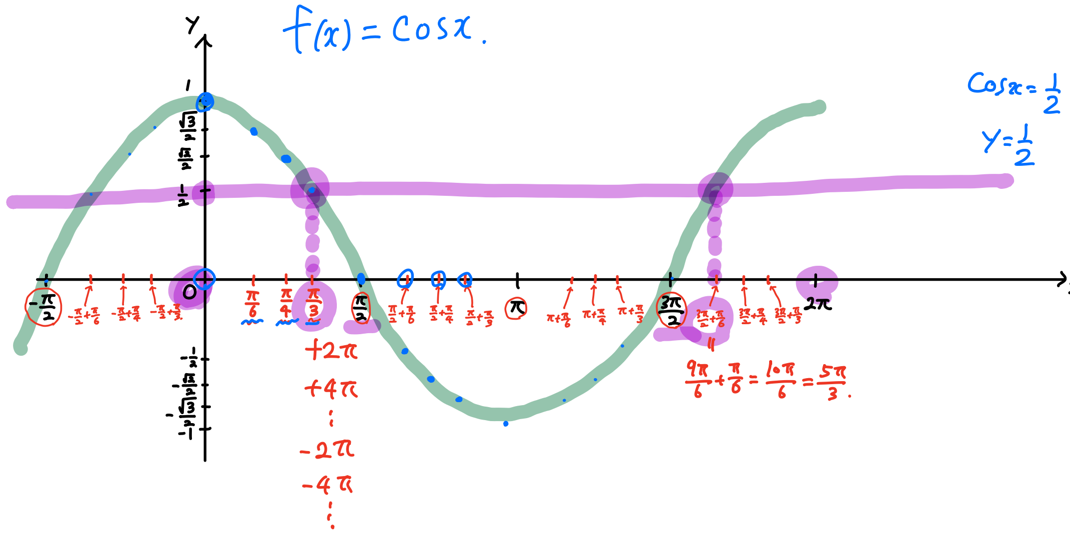
Graphs of $f(x) = \sin x$ and $f(x) = \cos x$.





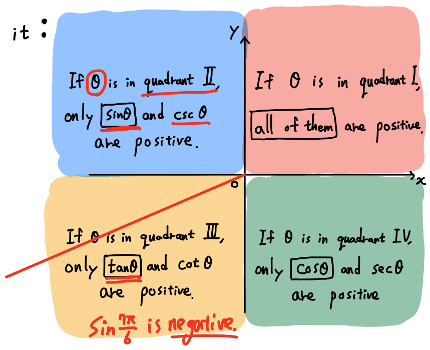
24. Find $\sin\frac{19\pi}{6}$.

Theorem on Reference Angles (10/29)

If θ is a nonquadrantal angle in standard position, then to find the value of a trigonometric function at θ find its value for the reference angle θ_R and prefix the appropriate sign.

Q. How to prefix the sign?

A. Use it:

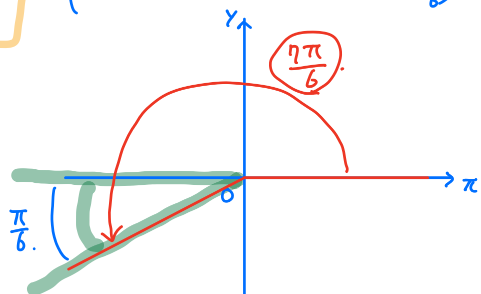


✓
$*\ \sin(x+2\pi)=\sin x.$

$\sin\frac{19\pi}{6}=\sin\left(\frac{7\pi}{6}+2\pi\right)=\sin\frac{7\pi}{6}$

$\left(\frac{19}{6}=\frac{12}{6}+\frac{7}{6}\Rightarrow\frac{19\pi}{6}=\frac{12\pi}{6}+\frac{7\pi}{6}=2\pi+\frac{7\pi}{6}\right)$

$\frac{7\pi}{6}=\frac{6\pi}{6}+\frac{\pi}{6}=\pi+\frac{\pi}{6}.$



$\theta=\frac{7\pi}{6}.\quad \theta_R=\frac{\pi}{6}.$

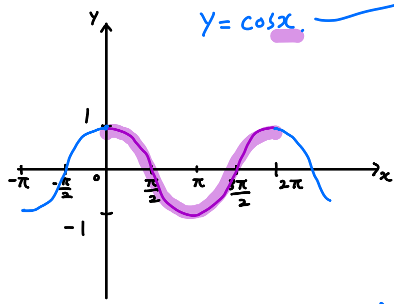
$\sin\frac{7\pi}{6}$ and $\sin\frac{\pi}{6}$ are either the same or only differ by the sign.

negative! $\quad \frac{1}{2}$.

"$\boxed{\sin\frac{7\pi}{6}=-\frac{1}{2}}$"

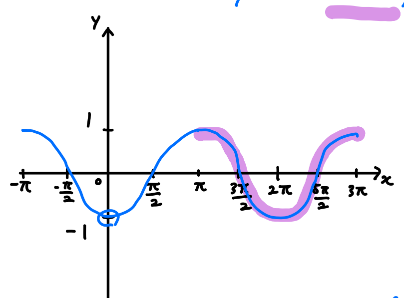
25. Draw a graph of $y=3\cos(x-\pi)+1$.

$y=\cos x$.



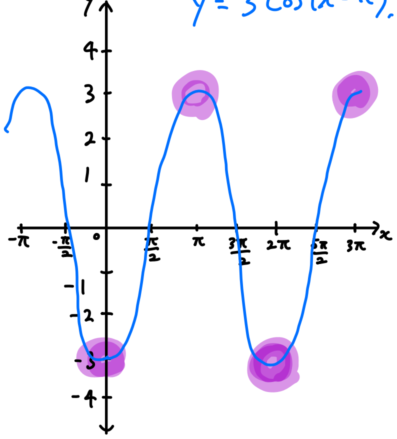
shift the graph π units to the right

$y=\cos(x-\pi)$.



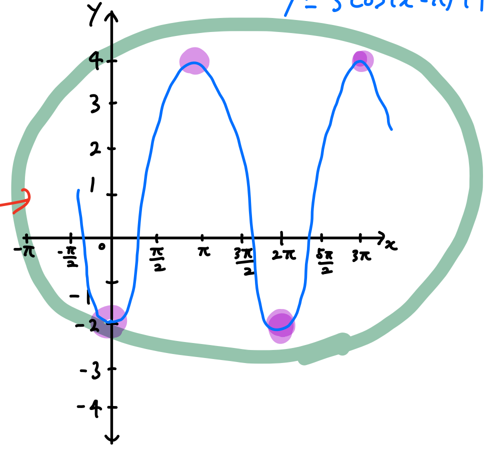
stretch the graph horizontally by a factor 3.

$y=3\cos(x-\pi)$.



shift the graph 1 unit above.

$y=3\cos(x-\pi)+1$.



26. Verify the identity : $\csc\theta - \sin\theta = \cot\theta\cos\theta$.

Fundamental Identities (10/25)

(1) The reciprocal identities : $\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

(2) The tangent and cotangent identities : $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

(3) The Pythagorean identities : ① $\sin^2\theta + \cos^2\theta = 1$ $\xrightarrow{-\sin^2\theta}$ $\boxed{\cos^2\theta = 1 - \sin^2\theta}$

② $1 + \tan^2\theta = \sec^2\theta$

③ $1 + \cot^2\theta = \csc^2\theta$

$$\csc\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta = \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta} = \boxed{\frac{1-\sin^2\theta}{\sin\theta}}.$$

$$\cot\theta\cos\theta = \frac{\cos\theta}{\sin\theta}\cdot\cos\theta = \frac{\cos^2\theta}{\sin\theta} = \boxed{\frac{1-\sin^2\theta}{\sin\theta}}.$$

the same. ✓

Hence $\csc\theta - \sin\theta = \cot\theta\cos\theta$ is an identity.

$\frac{1}{2}\sin 2x - \cos x = 0.$

$\frac{1}{\cancel{2}}\cdot\cancel{2}\cdot\sin x\cdot\cos x - \cos x = 0.$

27. Solve the equation $\sin x\cos x - \cos x = 0$, $x \in [0, 2\pi]$.

it means x lies in the interval $[0, 2\pi]$.

(* Double angle formula for sine : $\sin 2x = 2\sin x\cos x$.)

$\sin x\cos x - \cos x = 0.$

$\sin x\cos x - 1\cdot\cos x = 0.$

$\cos x\cdot(\sin x - 1) = 0.$

$\downarrow$ Z.F.T.

$\cos x = 0$ or $\sin x - 1 = 0$

$\downarrow$

$\cos x = 0$ or $\sin x = 1$. use the graph of sine.

use the graph of cosine.

$\Downarrow$ $\quad$ $\Downarrow$

$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ $\quad$ $x = \frac{\pi}{2}$.

$\Rightarrow$ $\boxed{x = \frac{\pi}{2} \text{ or } x = \frac{3\pi}{2}}$

28. Solve (1) $\cos\theta = 2$: $\boxed{\text{No solution}}$

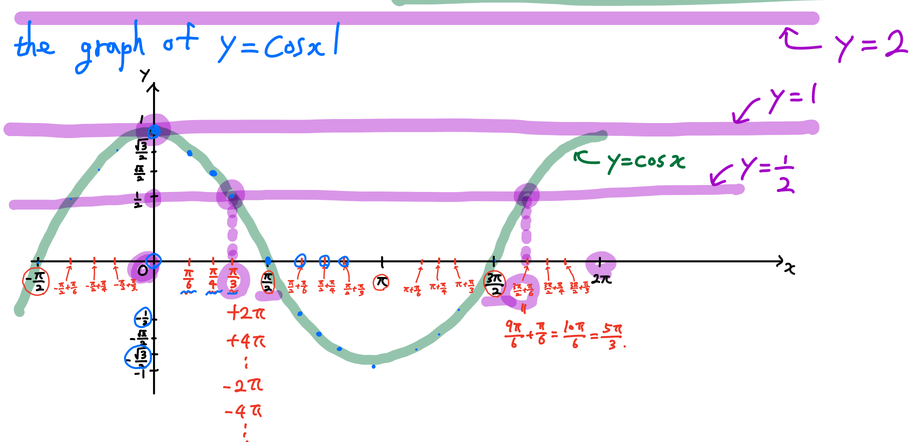
(2) $\cos\theta = 1$: $\theta = -4\pi, -2\pi, 0, 2\pi, 4\pi, \cdots$

$\boxed{\theta = 2\pi \cdot n \text{ for every integer } n.}$

(3) $\cos\theta = \frac{1}{2}$: $\theta = \cdots, \frac{\pi}{3}-2\pi, \frac{5\pi}{3}-2\pi, \frac{\pi}{3}, \frac{5\pi}{3}, \frac{\pi}{3}+2\pi, \frac{5\pi}{3}+2\pi, \cdots$

$\boxed{\theta = \frac{\pi}{3}+2\pi n, \frac{5\pi}{3}+2\pi n \text{ for every integer } n.}$

* Use the graph of $y = \cos x$!



29. Solve for θ the equation $-2\sin^2\theta - 5\cos\theta + 4 = 0$.

(Hint : use the identity $\sin^2\theta + \cos^2\theta = 1$ and the substitution $x = \cos\theta$.)

$$\left(\begin{array}{l} -\cos^2\theta \quad -\cos^2\theta \\ \sin^2\theta = 1 - \cos^2\theta. \end{array} \right)$$

$-2\underbrace{\sin^2\theta}_{(=1-\cos^2\theta)} - 5\cos\theta + 4 = 0.$

$-2(1-\cos^2\theta) - 5\cos\theta + 4 = 0.$

$-2 + 2\cos^2\theta - 5\cos\theta + 4 = 0.$

$2\cos^2\theta - 5\cos\theta + 2 = 0.$

$\downarrow$ replace $\cos\theta$ by x.

$2x^2 - 5x + 2 = 0.$

$(2x-1)(x-2) = 0. \xrightarrow{\text{Z.F.T.}} 2x-1=0$ or $x-2=0$.

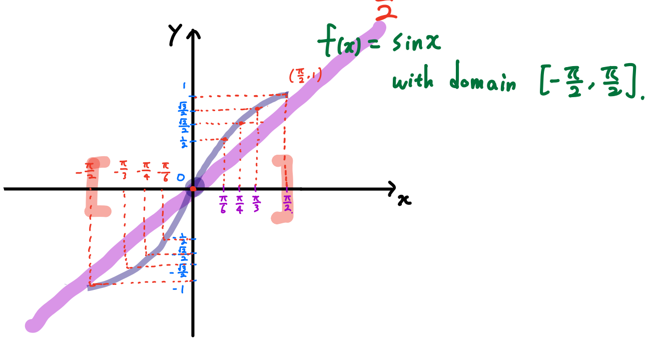
$\boxed{x = \frac{1}{2} \text{ or } x = 2.}$

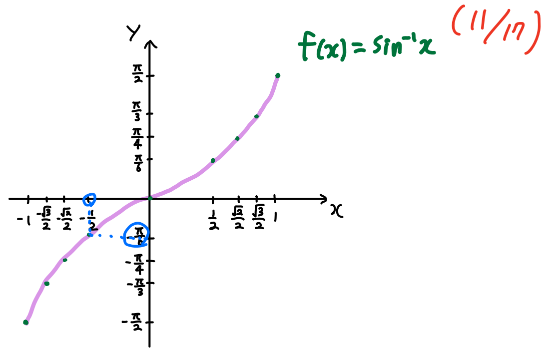
$\xrightarrow{\text{replace } x \text{ by } \cos\theta.}$ $\cos\theta = \frac{1}{2}$ or $\cos\theta = 2$. } by #28.

by #28. $\frac{\pi}{3}+2\pi n, \frac{5\pi}{3}+2\pi n$ for every integer n. ~~No solution~~.

$\Rightarrow \boxed{\theta = \frac{\pi}{3}+2\pi n, \frac{5\pi}{3}+2\pi n \text{ for every integer } n.}$

30. Find $\sin^{-1}\left(\sin \frac{7\pi}{6}\right)$ $\quad \frac{7\pi}{6} = \pi + \frac{\pi}{6}$. $\quad \sin\frac{7\pi}{6} = -\frac{1}{2}$





$$\sin^{-1}\left(\underbrace{\sin \frac{7\pi}{6}}_{-\frac{1}{2}}\right) = \sin^{-1}\left(-\frac{1}{2}\right) = \boxed{-\frac{\pi}{6}}.$$

31. Make the trigonometric substitution $x = a\tan\theta$ for $0 < \theta < \frac{\pi}{2}$ and $a > 0$. Simplify the resulting expression: $\sqrt{a^2 + x^2}$
(Hint: use the identity $1 + \tan^2\theta = \sec^2\theta$)

$\cos\theta > 0$.

$\sec\theta = \frac{1}{\cos\theta} > 0$.

$$\sqrt{a^2 + x^2} = \sqrt{a^2 + (a\tan\theta)^2} = \sqrt{a^2 + a^2\tan^2\theta} = \sqrt{a^2(1+\tan^2\theta)}$$

$$= \sqrt{a^2 \cdot \sec^2\theta}$$

$$= \boxed{a \cdot \sec\theta}$$

(because $a\sec\theta > 0$,
$\sqrt{a^2\sec^2\theta} = \sqrt{(a\sec\theta)^2} = a\sec\theta$)