Help Session : Today, Tomorrow 6–8pm
on Zoom !
(my Zoom meeting room!)

12. Find the equation of a circle centered at (3, 6) and passing through P(7, 3).

a center : (a, b)

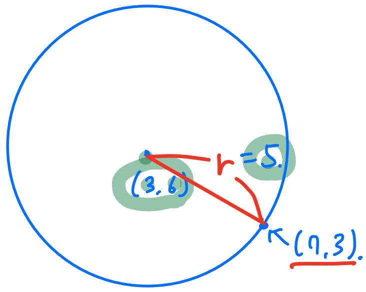
a radius : r

$\Downarrow$

$(x-a)^2+(y-b)^2=r^2$

distance between (x_1, y_1) and (x_2, y_2)

$\Rightarrow \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$



$r=\sqrt{(7-3)^2+(3-6)^2}$

$=\sqrt{4^2+(-3)^2}$

$=\sqrt{16+9}$

$=\sqrt{25}$

$=5.$

$\boxed{(x-3)^2+(y-6)^2=25}$

13. Find the quotient $Q(x)$ and the remainder $R(x)$ if the polynomial $4x^3+2x^2-x+5$ is divided by $x-1$.

$Q(x)$

$$\begin{array}{r l}
 & 4x^2+6x+5 \\
x-1 \,\big)\!\! & 4x^3+2x^2-x+5 \\
- & \boxed{4x^3-4x^2} \\
 & 6x^2-x \\
- & \boxed{6x^2-6x} \\
 & 5x+5 \\
- & \boxed{5x-5} \\
 & 10
\end{array}$$

$R(x)$.

$(6x^2-x)-(6x^2-6x)$

$4x^2$

$\square \times x = 4x^3$

$4x^2(x-1)=4x^3-4x^2$

$(4x^3+2x^2)-(4x^3-4x$

$2x^2-(-4x^2)=6x^2$

$\square \times x = 6x^2$

$\square \times x = 5x$

$\boxed{\begin{array}{l} Q(x)=4x^2+6x+5 \\ R(x)=10. \end{array}}$

14. Solve the inequality : $\dfrac{x-4}{x^2-4} > 0$

$\dfrac{x-4}{x^2-4} > 0$

$\dfrac{x-4}{x^2-2^2} > 0.$

$\dfrac{x-4}{(x+2)(x-2)} > 0.$

$x+2=0$, $\boxed{x=-2}$

$x-2=0$, $\boxed{x=2}$

$x-4=0$, $\boxed{x=4}$

		-2		2		4	
$\dfrac{x-4}{(x+2)(x-2)}$	$-$		$+$		$-$		$+$
$x+2$	$-$	0	$+$		$+$		$+$
$x-2$	$-$		$-$	0	$+$		$+$
$x-4$	$-$		$-$		$-$	0	$+$

$(-2,2)$ $(4,\infty)$

$\boxed{(-2,2)\cup(4,\infty)}$

15. Find the slope of the line through $X(-2,5)$ and $Y(3,7)$.

(x_2, y_2) (x_1, y_1)

(x_1, y_1) (x_2, y_2)

The slope of the line through (x_1, y_1) and (x_2, y_2)

$\Rightarrow \boxed{\dfrac{y_2-y_1}{x_2-x_1}}$

$\dfrac{7-5}{3-(-2)} = \boxed{\dfrac{2}{5}}.$

$\dfrac{5-7}{-2-3} = \dfrac{-2}{-5} = \boxed{\dfrac{2}{5}}$

16. Find the domain of the function $f(x) = \dfrac{1}{\sqrt{x-2}} - \sqrt{5-x}$.

$\sqrt{\text{negative}}$: ✗

$\dfrac{1}{0}$: ✗

$x-2 \geq 0.$

$x-2 \neq 0.$

$\boxed{x-2>0}$

$x>2$

$2<x$

$\boxed{5-x\geq 0}$

$+x \quad +x$

$5 \geq x$

$x \leq 5$

$\boxed{2<x\leq 5}$, $\boxed{(2,5]}$

17. Find the standard equation of the parabola that has a vertical axis and satisfies the given condition : Vertex $(2, -7)$, x-intercept 4.

$(4, 0)$.

The standard equation of the parabola that has a vertical axis with the vertex (h, k) is

$y = a(x-h)^2 + k$

$y = a(x-2)^2 - 7$.

$0 \quad 4$

$0 = a(4-2)^2 - 7$.

$0 = a \cdot 2^2 - 7$

$0 = 4a - 7$

$4a = 7$,

$a = \frac{7}{4}$.

$$\boxed{y = \frac{7}{4}(x-2)^2 - 7.}$$

18. Simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ using the function $f(x) = 2x^2 + x + 1$.

$x+h \quad x+h \quad x+h$

$f(x+h) = ?$

$f(x+h) = 2(x+h)^2 + (x+h) + 1$.

$= 2(x^2 + 2xh + h^2) + x + h + 1$

$= 2x^2 + 4xh + 2h^2 + x + h + 1$

$$\frac{f(x+h) - f(x)}{h} = \frac{(\cancel{2x^2} + 4xh + 2h^2 + \cancel{x} + h + \cancel{1}) - (\cancel{2x^2} + \cancel{x} + \cancel{1})}{h}$$

$$= \frac{4xh + 2h^2 + h}{h}$$

$$= \frac{\cancel{h}(4x + 2h + 1)}{\cancel{h}} = \frac{4x + 2h + 1}{1}$$

$$= \boxed{4x + 2h + 1}$$

19. Solve the equation $\log_5 (3x+10) = 2\log_5 x$.

positive positive

1) $n \cdot \log_a x = \log_a x^n$

2) $\log_a x = \log_a y \Rightarrow x = y$

$$\log_5 (3x+10) = \log_5 x^2$$

$$\Downarrow$$

$3x+10 = x^2$, $x^2 - 3x - 10 = 0$.

$-3x-10 \quad -3x-10$

$(x-5)(x+2) = 0$.

$x-5=0$ or $x+2=0$

$\boxed{x=5}$ or $\cancel{x=-2}$

$x=5$: $\log_5 (\overset{15}{3 \cdot 5}+10) = 2 \cdot \log_5 5$

$\log_5 25 = 2\log_5 5$

$\log_5 5^2$

$x=-2$: $\log_5 (3\cdot(-2)+10) = 2 \cdot \log_5 -2$: X

20. Solve the equation $3^{11x+5} = 9^{7x+2}$.

1) $(a^m)^n = a^{mn}$

2) $a^x = a^y \Rightarrow x = y$.

$$3^{11x+5} = (3^2)^{7x+2}$$

$$3^{11x+5} = 3^{2\cdot(7x+2)} \Rightarrow 11x+5 = 2\cdot(7x+2)$$

$11x+5 = 14x+4$.

$-11x \quad -11x$

$5 = 3x+4$

$-4 \quad -4$

$1 = 3x$. $\boxed{x = \frac{1}{3}}$.

21. Simplify $\dfrac{\ln(e^9 e^{-3y})}{3-y} = \dfrac{\ln(e^{9-3y})}{3-y} = \dfrac{9-3y}{3-y}$

$$= \frac{3\cancel{(3-y)}}{\cancel{3-y}}$$

$$= \frac{3}{1}$$

$$= \boxed{3}$$

1) $a^x \cdot a^y = a^{x+y}$

2) $\log_a (a^x) = x$.

$a=e \Rightarrow \log_e (e^x) = x$.

$\boxed{\ln(e^x) = x}$ ✓