- Final Exam : December 13 at 3pm – 5pm in $\boxed{\text{Ballentine Hall room 310.}}$

Total 20 problems.

- 3 or 4 problems are from the Special HW.
- 4 problems are sentence problems.
- Please do "Online Course Questionnaire (OCQ).
- Help Session for the Final : 12/7 ~ 12/9, 6pm – 8pm. Rawles Hall Room 104.

1. Find $\sqrt[3]{\dfrac{\sqrt[4]{13^{16}}}{13^7}}$

$$\frac{\sqrt[4]{13^{16}}}{13^7} = \frac{\left(13^{16}\right)^{\frac{1}{4}}}{13^7} = \frac{13^{16\cdot\frac{1}{4}}}{13^7} = \frac{13^4}{13^7} = \frac{1}{13^3}.$$

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$(a^n)^m = a^{mn}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[3]{\frac{\sqrt[4]{13^{16}}}{13^7}} = \sqrt[3]{\frac{1}{13^3}} = \frac{\sqrt[3]{1}}{\sqrt[3]{13^3}} = \boxed{\frac{1}{13}}$$

2. Simplify $\dfrac{(x+y)^2+(x-y)^2}{x^2+y^2} = \dfrac{(x^2+\cancel{2xy}+y^2)+(x^2-\cancel{2xy}+y^2)}{x^2+y^2}$

$(x+y)^2 = x^2+2xy+y^2$

$(x-y)^2 = x^2-2xy+y^2$

$= \dfrac{2x^2+2y^2}{x^2+y^2}$ You can go directly.

$= \dfrac{2\cancel{(x^2+y^2)}}{\cancel{x^2+y^2}} = \dfrac{2}{1} = \boxed{2}$

3. Solve $T = 2\pi\sqrt{\dfrac{l}{g}}$ for l : isolating l!

$\downarrow \div 2\pi$

$\dfrac{T}{2\pi} = \sqrt{\dfrac{l}{g}}$

$\downarrow$ take the square.

$\left(\dfrac{T}{2\pi}\right)^2 = \left(\sqrt{\dfrac{l}{g}}\right)^2, \quad \dfrac{T^2}{4\pi^2} = \dfrac{l}{g}. \xrightarrow{\text{multiply } g} \dfrac{gT^2}{4\pi^2} = l.$

$\boxed{l = \dfrac{gT^2}{4\pi^2}}$

4. Solve for y the equation $|y-3| = 5$

$\uparrow x \quad \uparrow a$

When a is positive number,

$|x| = a \Rightarrow x = a$ or $x = -a$

$|y-3| = 5 \Rightarrow y-3=5$ or $y-3=-5$

$\Rightarrow y = 5+3$ or $y=-5+3 \Rightarrow \boxed{y=8 \text{ or } y=-2}$

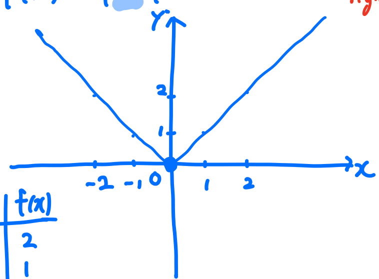
5. Factor the polynomial $3xy - xz + 6yw - 2zw$

Even terms $\Rightarrow$ Use grouping!

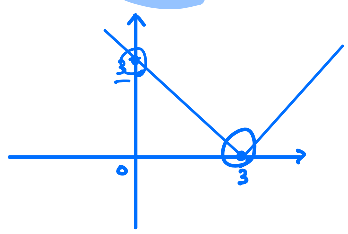
$$3xy - xz + 6yw - 2zw = (3xy - xz) + (6yw - 2zw)$$
$$= x(3y - z) + 2w(3y - z)$$
$$= \boxed{(3y - z)\cdot(x + 2w)}$$

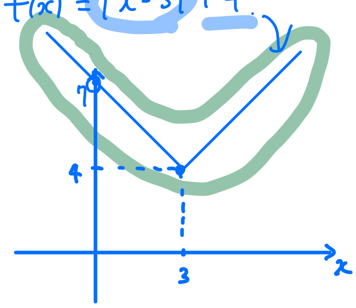
6. Draw the graph of the function $f(x) = |x - 3| + 4$

$f(x) = |x|$ shift 3 units to the right. $f(x) = |x - 3|$ shift 4 units above $f(x) = |x - 3| + 4$



x	$f(x)$
-2	2
-1	1
0	0
1	1
2	2





7. Find the equation of the circle that satisfies the following condition

: Endpoints of a diameter $A(7, 9)$ and $B(1, 1)$

If a circle has a center (a, b) $(4, 5)$

and a radius r $=5$

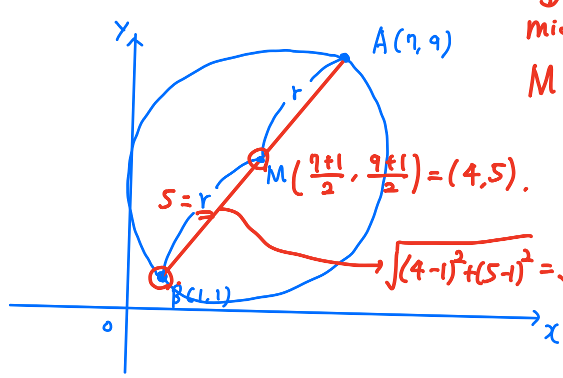
: $(x-a)^2 + (y-b)^2 = r^2$

$$\boxed{(x-4)^2 + (y-5)^2 = 25}$$

distance is $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$X = (x_1, y_1)$, $Y = (x_2, y_2)$

midpoint $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



$M\left(\frac{7+1}{2}, \frac{9+1}{2}\right) = (4, 5)$.

$\sqrt{(4-1)^2 + (5-1)^2} = \sqrt{9+16} = \sqrt{25}$
$=5$

8. Simplify $\dfrac{(x+y)(x-y)+(x-y)^2}{x^2-xy} = \dfrac{(x-y)\{(x+y)+(x-y)\}}{x(x-y)}$ (Proof 1)

$= \dfrac{\cancel{(x-y)}\cdot 2\cancel{x}}{\cancel{x}\cdot\cancel{(x-y)}} = \dfrac{2}{1} = \boxed{2}$

or Proof 2

$\dfrac{(x+y)(x-y)+(x-y)^2}{x^2-xy} = \dfrac{x^2-\cancel{y^2}+x^2-2xy+\cancel{y^2}}{x^2-xy} = \dfrac{2x^2-2xy}{x^2-xy} = \dfrac{2\cancel{x}\cancel{(x-y)}}{\cancel{x}\cancel{(x-y)}} = \boxed{2}$

$= \cdots$

9. Factor the polynomial $3y^2-9y-12$ $= 3(y+1)(y-4)$

(g.c.f is 3.)

$-4 = 4\times(-1)$
$= 2\times(-2)$
$= 1\times(-4)$

$= 3(1\cdot y^2-3y-4) = \boxed{3(y+1)(y-4)}$

Find two numbers whose sum is -3, and the product is -4.

$\boxed{\text{* } (3y+3)(y-4) \text{ is not a complete answer because } 3y+3 \text{ can be further factored.}}$

10. Let $B\neq 0$. Solve for y the equation $\dfrac{B^6y-2B^7}{B^6}-5 = 3-2B$

$\dfrac{P-Q}{R} = \dfrac{P}{R} - \dfrac{Q}{R}$

$\dfrac{B^6y-2B^7}{B^6} - 5 = 3-2B.$

$\dfrac{\cancel{B^6}y}{\cancel{B^6}} - \dfrac{2\cancel{B^6}B}{\cancel{B^6}} - 5 = 3-2B.$

$y-\cancel{2B}-5 = 3-\cancel{2B}.$

$y-5=3,$ $\boxed{y=8}$

11. Find the slope-intercept equation of the line ① passing through the point $P(0, -3)$ and ② parallel to the line $\ell : 2x - 5y + 130 = 0$.

$y = \underset{\frac{2}{5}}{m}x + \underset{-3}{b}$, where m is a slope, and b is a y-intercept.

$\boxed{\frac{2}{5}}$

$2x - \cancel{5y} + 130 = 0$

$\quad \cancel{+5y} \qquad +5y$

$2x + 130 = 5y$

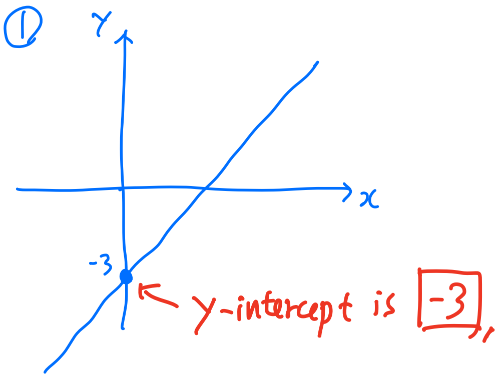
$\downarrow \div 5$

$\frac{2}{5}x + \frac{130}{5} = y$

$y = \frac{2}{5}x + \frac{130}{5}$.

slope.

①



Hence, $\boxed{y = \frac{2}{5}x - 3}$

* If two nonvertical lines are parallel, then they have the same slope!