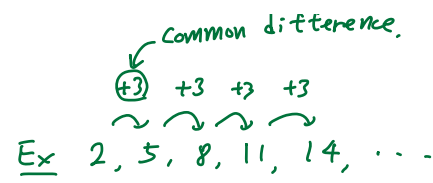


Section 10.2 Continue



$$= a_1 + a_2 + \dots + a_n$$

The partial sum S_n of an arithmetic sequence $a_1, a_2, \dots, a_n, \dots$ with common difference d satisfies the following theorem:

$$S_n = \frac{n(2a_1 + (n-1)d)}{2} \quad \text{or} \quad S_n = \frac{n(a_1 + a_n)}{2}$$

n terms

$$S_n = a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n$$

$a_k = a_1 + (k-1)d$

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \dots + (a_1 + (n-2)d) + (a_1 + (n-1)d)$$

$$S_n = a_n + a_{n-1} + a_{n-2} + \dots + a_2 + a_1$$

$$S_n = (a_1 + (n-1)d) + (a_1 + (n-2)d) + (a_1 + (n-3)d) + \dots + (a_1 + d) + a_1$$

$$+ S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \dots + (a_1 + (n-2)d) + (a_1 + (n-1)d)$$

$$2S_n = (2a_1 + (n-1)d) + (2a_1 + (n-1)d) + \dots + (2a_1 + (n-1)d) + (2a_1 + (n-1)d)$$

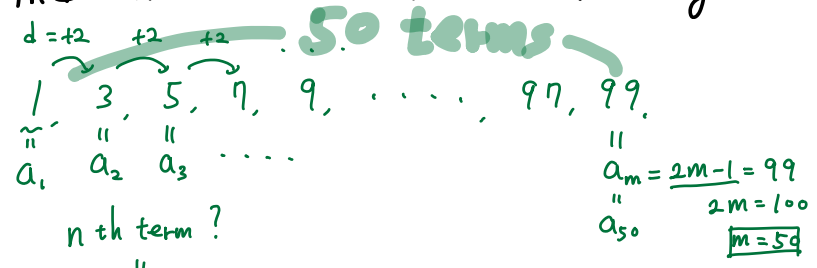
n terms

$$2S_n = n \cdot (2a_1 + (n-1)d)$$

$$\downarrow \div 2$$

$$S_n = \frac{n(2a_1 + (n-1)d)}{2} = \frac{n(a_1 + a_1 + (n-1)d)}{2} = \frac{n(a_1 + a_n)}{2}$$

Ex Find the sum of all the odd integers from 1 through 99.



$$a_n = a_1 + (n-1)d = 1 + (n-1) \cdot 2 = 1 + 2n - 2 = 2n - 1$$

$$1 + 3 + 5 + \dots + 97 + 99 = \frac{50 \cdot (1 + 99)}{2} = \frac{50 \cdot 100}{2} = \frac{5000}{2} = 2500$$

50 terms

* arithmetic mean of a and b : $\frac{a+b}{2}$

[a $\frac{a+b}{2}$ b : arithmetic sequence!] Ex $4 \xrightarrow{+2} \boxed{6} \xrightarrow{+2} 8$

* k arithmetic mean of a and b : k numbers C_1, C_2, \dots, C_k such that $a, C_1, C_2, \dots, C_k, b$ form an arithmetic sequence.

Ex. 2 arithmetic mean of 4 and 10 are $\boxed{6 \text{ and } 8}$: $4 \xrightarrow{+2} \boxed{6} \xrightarrow{+2} \boxed{8} \xrightarrow{+2} 10$

Ex Insert four arithmetic means between 3 and 18 : $\boxed{6, 9, 12, 15}$

Then C_1, C_2, C_3, C_4

$3 \xrightarrow{+d=3} \underbrace{C_1=6}_{\frac{3+d}{1}} \xrightarrow{+d=3} \underbrace{C_2=9}_{\frac{3+2d}{1}} \xrightarrow{+d=3} \underbrace{C_3=12}_{\frac{3+3d}{1}} \xrightarrow{+d=3} \underbrace{C_4=15}_{\frac{3+4d}{1}} \xrightarrow{+d=3} \underbrace{18}_{\frac{3+5d}{1}}$: arithmetic sequence!

$\hookrightarrow 3+5d=18$
 $5d=15, \boxed{d=3}$

Section 10.3 Geometric Sequences.

A sequence $a_1, a_2, \dots, a_n, \dots$ is a geometric sequence if $a_1 \neq 0$.

and if there is a real number $r \neq 0$ such that

$$\boxed{a_{k+1} = a_k r} \text{ for every positive integer } k.$$

The number $r = \frac{a_{k+1}}{a_k}$ is called the common ratio of the sequence.

Ex $2 \xrightarrow{\times 2} 4 \xrightarrow{\times 2} 8 \xrightarrow{\times 2} 16 \xrightarrow{\times 2} 32, \dots : r=2.$

Ex 1) $2 \xrightarrow{\times 3} 6 \xrightarrow{\times 3} 18 \xrightarrow{\times 3} 54, \dots, 2 \cdot 3^{n-1}, \dots : r=3.$

2) $8 \xrightarrow{\times (-\frac{1}{2})} -4 \xrightarrow{\times (-\frac{1}{2})} 2 \xrightarrow{\times (-\frac{1}{2})} -1, \dots, 8 \cdot (-\frac{1}{2})^{n-1}, \dots : r = -\frac{1}{2}.$

If the common ratio is r ,

$$a_1 \xrightarrow{\times r} a_2 \xrightarrow{\times r} a_3 \xrightarrow{\times r} a_4 \dots \dots \boxed{\begin{matrix} a_n \\ \parallel \\ a_1 \cdot r^{n-1} \end{matrix}}$$

$$\Rightarrow \underline{a_n = a_1 \cdot r^{n-1}}$$

Ex The second term of a geometric sequence is 6,
 and the fifth term is $\frac{3}{4}$. Find the seventh term.

$$\begin{aligned} a_1 \cdot r^1 &= a_2 = 6 \\ a_1 \cdot r^4 &= a_5 = \frac{3}{4} \\ a_7 &=? \end{aligned}$$

$$* a_n = a_1 \cdot r^{n-1}$$

$$\begin{aligned} a_1 r &= 6 \\ a_1 r^4 &= \frac{3}{4} \end{aligned}$$

$$\Rightarrow \frac{a_1 r}{a_1 r^4} = \frac{6}{\frac{3}{4}}$$

$$\Rightarrow \frac{1}{r^3} = \frac{2 \cdot 4}{1 \cdot 3} = 8 \quad \frac{1}{r^3} = 8 \quad r^3 = \frac{1}{8} = \left(\frac{1}{2}\right)^3$$

$$r = \frac{1}{2}$$

$$a_1 r = 6 \Rightarrow a_1 \cdot \frac{1}{2} = 6 \quad a_1 = 12$$

$$\begin{aligned} a_7 &= a_1 \cdot r^6 = 12 \cdot \left(\frac{1}{2}\right)^6 \\ &= 12 \cdot \frac{1}{2^6} \\ &= 12 \cdot \frac{1}{64} = \frac{3}{16} \end{aligned}$$

The partial sum S_n of a geometric sequence $a_1, a_2, \dots, a_n, \dots$ with common ratio r satisfies the following theorem:

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_n = a_1 + a_2 + a_3 + \dots + a_n$$

$$a_k = a_1 \cdot r^{k-1}$$

$$S_n = a_1 + a_1 r + a_1 r^2 + \dots + a_1 r^{n-1}$$

$\downarrow \times r$

$$r \cdot S_n = r \cdot (a_1 + a_1 r + a_1 r^2 + \dots + a_1 r^{n-1})$$

$$r \cdot S_n = a_1 r + a_1 r^2 + \dots + a_1 r^n$$

$$S_n = a_1 + a_1 r + a_1 r^2 + \dots + a_1 r^{n-1}$$

$$\underline{\quad} - \underline{\quad} \Rightarrow r \cdot S_n = a_1 r + a_1 r^2 + \dots + a_1 r^n$$

$$S_n - r \cdot S_n = a_1 - a_1 r^n \quad S_n(1-r) = a_1(1-r^n) \xrightarrow{:(1-r)} S_n = \frac{a_1(1-r^n)}{1-r}$$

Ex Find the sum of the first six terms of the
geometric sequence $2, 1, \frac{1}{2}, \dots$.

$$a_1 = 2, r = \frac{1}{2}, \text{ and } n = 6.$$

$$\begin{aligned} \text{Hence, } S_6 &= \frac{2 \cdot (1 - (\frac{1}{2})^6)}{1 - \frac{1}{2}} = \frac{2(1 - \frac{1}{2^6})}{\frac{1}{2}} = 4(1 - \frac{1}{2^6}) \\ &= 4(1 - \frac{1}{64}) \\ &= 4 \cdot \frac{63}{64} = \boxed{\frac{63}{16}} \end{aligned}$$

* geometric mean of a and b : \sqrt{ab}

$a \quad \sqrt{ab} \quad b$: geometric sequence!

Ex 3 and 12 : $\sqrt{3 \cdot 12} = \sqrt{36} = \underline{6}$ is the geometric mean.

$3 \xrightarrow{\times 2} 6 \xrightarrow{\times 2} 12$: geometric sequence!