Section 10.2 Continue

common difference

+3 +3 +3 +3

Ex 2, 5, 8, 11, 14, . . .

$= \boxed{a_1 + a_2 + \cdots + a_n}$

The partial sum S_n of an arithmetic sequence $a_1, a_2, \cdots, a_n, \cdots$ with common difference d satisfies the following theorem:

$$S_n = \frac{n(2a_1 + (n-1)d)}{2} \quad \text{or} \quad S_n = \frac{n(a_1 + a_n)}{2}$$

$$\boxed{a_k = a_1 + (k-1)d}$$

$$S_n = \underbrace{a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n}_{n \text{ terms}}$$

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-2)d) + (a_1 + (n-1)d)$$

$$S_n = a_n + a_{n-1} + a_{n-2} + \cdots + a_2 + a_1$$

$$\begin{array}{rl} S_n = & (a_1+(n-1)d) + (a_1+(n-2)d) + (a_1+(n-3)d) + \cdots + (a_1+d) + a_1 \\ + \; S_n = & a_1 + (a_1+d) + (a_1+2d) + \cdots + (a_1+(n-2)d) + (a_1+(n-1)d) \\ \hline 2S_n = & \underbrace{(2a_1+(n-1)d) + (2a_1+(n-1)d) + (\;''\;) + \cdots + (\;''\;) + (\;''\;)}_{n \text{ terms}} \end{array}$$

$$2S_n = n \cdot (2a_1 + (n-1)d)$$

$\downarrow \div 2$

$$S_n = \frac{n(2a_1 + (n-1)d)}{2} = \frac{n(a_1 + \overbrace{a_1 + (n-1)\cdot d}^{a_n})}{2} = \frac{n(a_1 + a_n)}{2}$$

Ex Find the sum of all the odd integers from 1 through 99.

$d = +2 \quad +2 \quad +2$ 50 terms

$1, 3, 5, 7, 9, \ldots, 97, 99.$

$a_1 \quad a_2 \quad a_3 \quad \ldots$

$a_m = 2m - 1 = 99$

$2m = 100$

$\boxed{m = 50}$

a_{50}

n th term ?

$$a_n = a_1 + (n-1)d = 1 + (n-1)\cdot 2 = 1 + 2n - 2$$

$$= \boxed{2n - 1}$$

$$\underbrace{1 + 3 + 5 + \cdots + 97 + 99}_{50 \text{ terms}} = \frac{50 \cdot (1 + 99)}{2} = \frac{50 \cdot 100}{2} = \frac{5000}{2} = \boxed{2500}$$

* arithmetic mean of a and b : $\frac{a+b}{2}$.

$\boxed{a \quad \frac{a+b}{2} \quad b : \text{arithmetic sequence!}}$ Ex $4 \underset{+2}{\frown} \boxed{6} \underset{+2}{\frown} 8$

* k arithmetic mean of a and b : k numbers $c_1, c_2, \cdots, c_k$

such that $a \; c_1 \; c_2 \; \cdots\cdots \; c_k \; b$ form an arithmetic sequence.

Ex. 2 arithmetic mean of 4 and 10 are $\boxed{6 \text{ and } 8}$: $\underline{4 \overset{+2}{\frown} \boxed{6} \overset{+2}{\frown} \boxed{8} \overset{+2}{\frown} 10}$

Ex Insert four arithmetic means between 3 and 18. : $\boxed{6, 9, 12, 15}$

c_1, c_2, c_3, c_4.

Then $3 \xrightarrow{+d=3} c_1 = 6 \xrightarrow{+d=3} c_2 = 9 \xrightarrow{+d=3} c_3 = 12 \xrightarrow{+d=3} c_4 = 15 \xrightarrow{+d=3} 18$: arithmetic sequence!

$c_1 = 3+d$, $c_2 = 3+2d$, $c_3 = 3+3d$, $c_4 = 3+4d$, $18 = 3+5d$

$3+5d = 18$.

$5d = 15$, $\boxed{d=3}$

Section 10.3 Geometric Sequences

A sequence $a_1, a_2, \cdots, a_n, \cdots$ is a geometric sequence

if $a_1 \neq 0$,

and if there is a real number $r \neq 0$ such that

$\boxed{a_{k+1} = a_k r}$ for every positive integer k.

The number "$\boxed{r = \frac{a_{k+1}}{a_k}}$" is called the common ratio of the sequence.

Ex $2 \xrightarrow{\times 2} 4 \xrightarrow{\times 2} 8 \xrightarrow{\times 2} 16 \xrightarrow{\times 2} 32, \cdots : r = 2$.

Ex 1) $2 \xrightarrow{\times 3} 6 \xrightarrow{\times 3} 18 \xrightarrow{\times 3} 54, \cdots, 2 \cdot 3^{n-1}, \cdots \quad : r = 3$.

2) $8 \xrightarrow{\times(-\frac{1}{2})} -4 \xrightarrow{\times(-\frac{1}{2})} 2 \xrightarrow{\times(-\frac{1}{2})} -1, \cdots, 8 \cdot (-\frac{1}{2})^{n-1}, \cdots \quad : r = -\frac{1}{2}$.

If the common ratio is r, $a_1 \xrightarrow{\times r} a_2 \xrightarrow{\times r} a_3 \xrightarrow{\times r} a_4 \cdots \boxed{a_n}$

$a_2 = a_1 r$, $a_3 = a_1 r^2$, $a_4 = a_1 r^3$, $\boxed{a_n = a_1 \cdot r^{n-1}}$

$$\Rightarrow a_n = a_1 \cdot r^{n-1}$$

a_2

Ex The second term of a geometric sequence is 6 and the fifth term is $\frac{3}{4}$. Find the seventh term.

a_5

$a_7 = a_1 \cdot r^6 = 12 \cdot \left(\frac{1}{2}\right)^6$

$a_1 \cdot r^1 = a_2 = 6$

$a_1 \cdot r^4 = a_5 = \frac{3}{4}$

$a_7 = ?$

$\boxed{* a_n = a_1 \cdot r^{n-1}}$

$a_1 r = 6$

$a_1 r^4 = \frac{3}{4}$

$\Rightarrow \frac{\cancel{a_1} r}{\cancel{a_1} r^4} = \frac{\frac{6}{1}}{\frac{3}{4}}$

$\Rightarrow \frac{1}{r^3} = \frac{\overset{2}{\cancel{6}} \cdot 4}{1 \cdot \cancel{3}_1} = 8. \quad \frac{1}{r^3} = 8. \quad r^3 = \frac{1}{8} = \left(\frac{1}{2}\right)^3$

$\boxed{r = \frac{1}{2}}$

$= 12 \cdot \frac{1}{2^6}$

$= \overset{3}{\cancel{12}} \cdot \frac{1}{\cancel{64}_{16}} = \boxed{\frac{3}{16}}$

$a_1 r = 6. \Rightarrow a_1 \cdot \frac{1}{2} = 6. \quad a_1 = 12.$

The partial sum S_n of a geometric sequence $a_1, a_2, \cdots, a_n, \cdots$ with common ratio r satisfies the following theorem:

$$S_n = \frac{a_1 (1 - r^n)}{1 - r}$$

$S_n = a_1 + a_2 + a_3 + \cdots + a_n$

$\boxed{a_k = a_1 \cdot r^{k-1}}$

$S_n = a_1 + a_1 \cdot r + a_1 \cdot r^2 + \cdots + a_1 \cdot r^{n-1}$

$\downarrow \times r$

$r \cdot S_n = r \cdot (a_1 + a_1 \cdot r + a_1 \cdot r^2 + \cdots + a_1 \cdot r^{n-1})$

$r \cdot S_n = a_1 \cdot r + a_1 \cdot r^2 + \cdots \cdots + a_1 \cdot r^n$

$S_n = a_1 + \cancel{a_1 r} + \cancel{a_1 r^2} + \cdots + \cancel{a_1 r^{n-1}}$

$- \; r \cdot S_n = \cancel{a_1 r} + \cancel{a_1 r^2} + \cdots + a_1 \cdot r^n$

$S_n - r S_n = a_1 - a_1 \cdot r^n. \quad S_n (1 - r) = a_1 (1 - r^n) \xrightarrow{\div (1-r)} \boxed{S_n = \frac{a_1 (1 - r^n)}{1 - r}}$

Ex Find the sum of the first six terms of the geometric sequence $2, 1, \frac{1}{2}, \cdots$.

$n=6$.

$a_1 = 2$, $r = \frac{1}{2}$, and $n = 6$.

Hence, $S_6 = \dfrac{2\cdot(1-(\frac{1}{2})^6)}{1-\frac{1}{2}} = \dfrac{2(1-\frac{1}{2^6})}{\frac{1}{2}} = 4(1-\frac{1}{2^6})$

$= 4(1-\frac{1}{64})$

$= 4 \cdot \frac{63}{64} = \boxed{\frac{63}{16}}$.

* geometric mean of a and b : $\sqrt{ab}$

$a \quad \sqrt{ab} \quad b$: geometric sequence!

Ex 3 and 12 : $\sqrt{3\cdot 12} = \sqrt{36} = 6$ is the geometric mean.

$3 \xrightarrow{\times 2} 6 \xrightarrow{\times 2} 12$: geometric sequence!