HW 11 : due today at 11:59pm. HW 12 : due next Friday.

Special HW : due 12/12 11:59 pm

Plan for the next week and today.

Today (12/3) : Section 10.1 , Section 10.2.

(next Sunday (12/5) : Will post practice exam on Canvas.)

next Monday (12/6) : Section 10.2, 10.3.

next Tuesday – Friday (12/7 ~ 12/10) : Will go over

Help Session : Tuesday ~ Thursday (12/7 – 12/9) : 6-8pm in RH 104 (Rawles Hall).

12/12 night, 12/13 moring : Office hour.

12/13 3pm : Exam.

# Section 10.1 Continued

① the first term is given.
and ② k+1 th term is expressed
in terms of k th term.

Sometimes, sequences are defined recursively!

**Ex** $a_1 = 5$, $a_{k+1} = 2a_k$ for $k \geq 1$.

$a_{1+1} = 2a_1 \Rightarrow a_2 = 2a_1 = 2\cdot 5$

$a_{2+1} = 2a_2 \Rightarrow a_3 = 2a_2 = 2\cdot 2\cdot 5 = 2^2\cdot 5$

$a_{3+1} = 2a_3 \Rightarrow a_4 = 2a_3 = 2\cdot 2^2\cdot 5 = 2^3\cdot 5$

$\vdots$

$$\begin{aligned} & a_1 = 5 \\ \cdot 2 \quad & a_2 = 2\cdot 5 \\ \cdot 2 \quad & a_3 = 2^2\cdot 5 \\ \cdot 2 \quad & a_4 = 2^3\cdot 5 \end{aligned}$$

$$a_1 \xrightarrow{\times 2} a_2 \xrightarrow{\times 2} a_3 \xrightarrow{\times 2} a_4 \cdots \xrightarrow{\times 2} a_n \Rightarrow a_n = a_1\cdot 2^{n-1}$$

$$= \boxed{5\cdot 2^{n-1}}$$

Sometimes, we need to consider the sum of the consecutive terms.

**Ex** $a_1 \quad a_2 \quad \boxed{a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7} \quad \cdots$

$a_3 + a_4 + a_5 + a_6 + a_7$.

We have "Summation notation" that shorten the notation!

## Summation Notation

$$\sum_{k=m}^{n} a_k = a_m + a_{m+1} + a_{m+2} + \cdots + a_n$$

$$S_n = \sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_n$$

$S_n$ ← n th partial sum.

Given a sequence $a_1, a_2, a_3, \cdots, a_n, \cdots$ ,

the sequence $S_1, S_2, S_3, \cdots, S_n, \cdots$ is called the sequence of partial sums.

$$\begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ \vdots \end{matrix} \longrightarrow \begin{matrix} a_1 = S_1 \\ a_1 + a_2 = S_2 \\ a_1 + a_2 + a_3 = S_3 \\ a_1 + a_2 + a_3 + a_4 = S_4 \\ \vdots \quad \vdots \end{matrix}$$

Ex) ↙ given sequence

$1, 2, 3, 4, 5, \cdots$

$\Downarrow$

$1,\ 1+2,\ 1+2+3,\ 1+2+3+4, \cdots$

$1,\ 3,\ 6,\ 10, \cdots$

↑ the sequence of partial sum.

Ex Find the first four terms and the nth terms of the sequence of partial sum associated with the sequence

$$1, 3, 5, \dots, \underset{n \uparrow th}{\underline{2n-1}}, \dots$$

$$\begin{array}{cccccccc} a_1 & a_2 & a_3 & a_4 & a_5 & & a_n & \\ 1, & 3, & 5, & 7, & 9, & \cdots, & 2n-1, & \cdots \end{array}$$

$$\begin{array}{cccccc} S_1 & S_2 & S_3 & S_4 & & S_n \\ 1, & 1+3, & 1+3+5, & 1+3+5+7, & \cdots, & 1+3+5+\cdots+(2n-1), \cdots \\ \| & \| & \| & \| & & \| ? \\ 1 & 4 & 9 & 16. & & n^2. \\ \| & \| & \| & \| & & \\ 1^2 & 2^2 & 3^2 & 4^2 & & \end{array}$$

$$S_n = \overbrace{1 + 3 + 5 + \cdots + (2n-3) + (2n-1)}^{n \text{ terms}}$$

$$+\ S_n = (2n-1) + (2n-3) + \cdots\cdots + 3 + 1$$

$$\boxed{2\cdot S_n} = \underbrace{2n + 2n + \quad + 2n + 2n}_{n \text{ terms}} = 2n \times n = \boxed{2n^2}$$

$$\boxed{S_n = n^2} \ //$$

Constant sequence $c, c, c, \dots, c, \dots$ satisfies the following theorem :

(1) $$\sum_{k=1}^{n} \overset{a_k}{c} = \overbrace{c + c + \cdots + c}^{n \text{ of them}} = \boxed{n\cdot c}$$

(2) $$\sum_{k=m}^{n} \overset{a_k}{c} = a_m + a_{m+1} + \cdots + a_n = \overbrace{c + c + \cdots + c}^{n-m+1} = \boxed{(n-m+1)\cdot c}$$

$$\overbrace{\underbrace{a_1, a_2, \cdots, a_{m-1}}_{m-1}, \underbrace{\boxed{a_m, a_{m+1}, \cdots, a_n}}_{n-(m-1) = n-m+1}}^{n}$$

Ex 1) $\sum_{k=1}^{5} \overset{a_k}{\overset{\shortparallel}{3}} = a_1+a_2+a_3+a_4+a_5 = 3+3+3+3+3 = 3\cdot5 = \boxed{15}$

2) $\sum_{k=5(m)}^{10(n)} \overset{a_k}{\overset{\shortparallel}{4}} = \underline{\overset{4}{a_5} + \overset{4}{a_6} + \cdots + \overset{4}{a_{10}}} = \underline{\overbrace{4+4+\cdots+4}^{6}} = 4\cdot 6 = \boxed{24}$

✓ 3) $\sum_{k=1}^{4} \underset{a_k}{\underset{\shortparallel}{(2k-1)}} = a_1+a_2+a_3+a_4 = 1+3+5+7 = \boxed{16}$

$$\begin{aligned} a_k = 2k-1 \Rightarrow a_1 &= 2\cdot1-1 = 2-1 = 1 \\ a_2 &= 2\cdot2-1 = 4-1 = 3 \\ a_3 &= 2\cdot3-1 = 6-1 = 5 \\ a_4 &= 2\cdot4-1 = 8-1 = 7 \\ &\vdots \end{aligned}$$

Given sequences $\begin{cases} a_1, a_2, \cdots, a_n, \cdots \\ b_1, b_2, \cdots, b_n, \cdots \end{cases}$ ,

the sum of two sequence is $a_1+b_1, a_2+b_2, \cdots, a_n+b_n, \cdots$

and the difference of two sequence is $a_1-b_1, a_2-b_2, \cdots, a_n-b_n, \cdots$.

A $\overset{c}{\overset{\shortparallel}{\text{constant}}}$ multiple of a sequence $a_1, a_2, \cdots, a_n, \cdots$

is $c\cdot a_1, c\cdot a_2, \cdots, c\cdot a_n, \cdots$.

They satisfy the following theorem : (1) $\sum_{k=1}^{n}(a_k+b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$,

(2) $\sum_{k=1}^{n}(a_k-b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$,

(3) $\sum_{k=1}^{n} c a_k = c\cdot \sum_{k=1}^{n} a_k$.

# Section 10.2 Arithmetic Sequences.

A sequence $a_1, a_2, \cdots, a_n, \cdots$ is an arithmetic sequence if there is a real number $d$ such that for every positive integer $k$,

$$a_{k+1} = a_k + d.$$

The number $d = a_{k+1} - a_k$ is called the common difference of the sequence.

Ex $\quad 1, \xrightarrow[+2]{} 3, \xrightarrow[+2]{} 5, \xrightarrow[+2]{} 7, \xrightarrow[+2]{} 9, \cdots, 2n-1, \cdots$ : 2 is the common difference.

Ex 1) $-2, \xrightarrow{+3} 1, \xrightarrow{+3} 4, \cdots, 3n-5, \cdots$ : $\boxed{d = 3}$.

2) $5, \xrightarrow[+(-2)]{} 3, \xrightarrow[+(-2)]{} 1, \cdots, -2n+7, \cdots$ : $\boxed{d = -2}$.

If a sequence $a_1, a_2, a_3, \dots, a_n$ is arithmetic with common difference $d$.

$$a_{k+1} - a_k = d.$$

$$a_{k+1} = a_k + d.$$

$$\left( \begin{array}{l} a_2 = a_1 + d \\ a_3 = a_2 + d \\ a_4 = a_3 + d \\ \vdots \end{array} \right)$$

$$a_1 = a_1$$

$$\underline{a_2} \overset{\checkmark}{=} \underline{a_1 + d}$$

$$\underline{a_3} \overset{\checkmark}{=} \underline{\underline{a_2}} + d = (a_1 + d) + d = \underline{\underline{a_1 + 2d}}$$

$$\underline{a_4} = \underline{\underline{a_3}} + d = (a_1 + 2d) + d = \underline{\underline{a_1 + 3d}}.$$

$$\vdots$$

$$" \boxed{a_n = a_1 + (n-1) \cdot d.} "$$

**Ex** The first three terms of an arithmetic sequence are $17, 14, 11$. Find the $\underline{\text{tenth term}}$.

$$17 \underbrace{,}_{+(-3)} 14 \underbrace{,}_{+(-3)} 11. \qquad "a_{10}.$$

↳ (common difference) $= d = -3$. $a_1 = 17$.

$$a_{10} = a_1 + (10-1) \cdot d = 17 + 9 \cdot (-3) = 17 + (-27)$$

$$= \boxed{-10}.$$

Ex If the $\overset{a_5}{\text{fifth term}}$ of an arithmetic sequence is 7 and the $\underset{a_{10}}{\text{tenth term}}$ is 22. Find the $\underset{a_7}{\text{seventh term}}$.

From the given conditions,

$$\begin{cases} a_1+4\cdot d = a_5 = 7, \\ a_1+9\cdot d = a_{10} = 22. \end{cases}$$

We solve the system of linear equations.

$$(a_1+4d)=7$$
$$(a_1+9d)=22$$
$$(a_1+4d)-(a_1+9d)=7-22.$$
$$-5d=-15.$$
$$\underline{d=3}.$$
$$a_1+4\cdot 3=7,\quad a_1+12=7,\quad \underline{a_1=-5}$$

Hence, $a_7 = a_1+6\cdot d$

$= -5+6\cdot 3$

$= -5+18$

$= \boxed{13}$