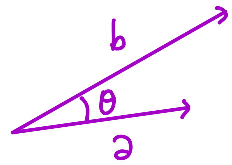
HWII : due tommorow! 11:59pm.

Section 8.4 Continued.

Recall!

Theorem on the Cosine of the Angle Between Vectors.

If θ is the angle between two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$, then



$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|\cdot\|\mathbf{b}\|}.$$

$\theta = \frac{\pi}{2}$.

Ex Show that the pair of vectors is orthogonal.

(a) i, j

$a = i = \langle 1, 0 \rangle$

$b = j = \langle 0, 1 \rangle$.

$a\cdot b = \langle 1, 0\rangle \cdot \langle 0, 1\rangle = 1\cdot 0 + 0\cdot 1$
$= 0$.

$\cos\theta = \frac{a\cdot b}{\|a\|\cdot\|b\|} = \frac{0}{\|a\|\cdot\|b\|} = 0$.

$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$.

Hence i and j are orthogonal!

(b) $3i-4j$, $8i+6j$.

$a = 3i-4j = \langle 3, -4\rangle$

$b = 8i+6j = \langle 8, 6\rangle$

$a\cdot b = \langle 3, -4\rangle\cdot\langle 8, 6\rangle = 3\cdot 8 + (-4)\cdot 6$
$= 24 + (-24)$
$= 0$.

$\cos\theta = \frac{a\cdot b}{\|a\|\cdot\|b\|} = \frac{0}{\|a\|\cdot\|b\|} = 0$.

$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$.

Hence $3i-4j$ and $8i+6j$ are orthogonal!

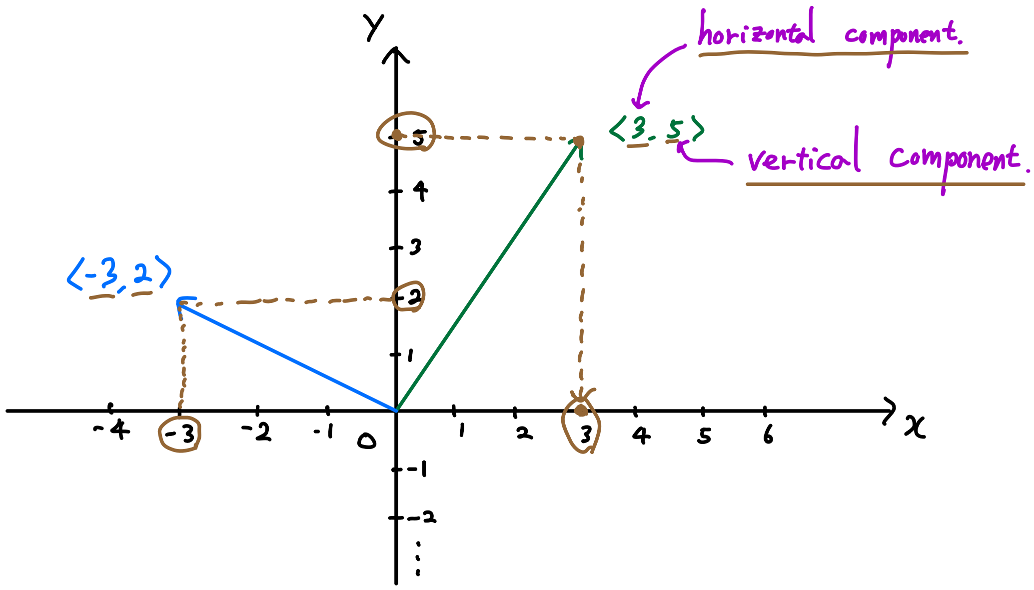
Recall!

$\vec{i} = \langle 1, 0 \rangle$, $\vec{j} = \langle 0, 1 \rangle$.

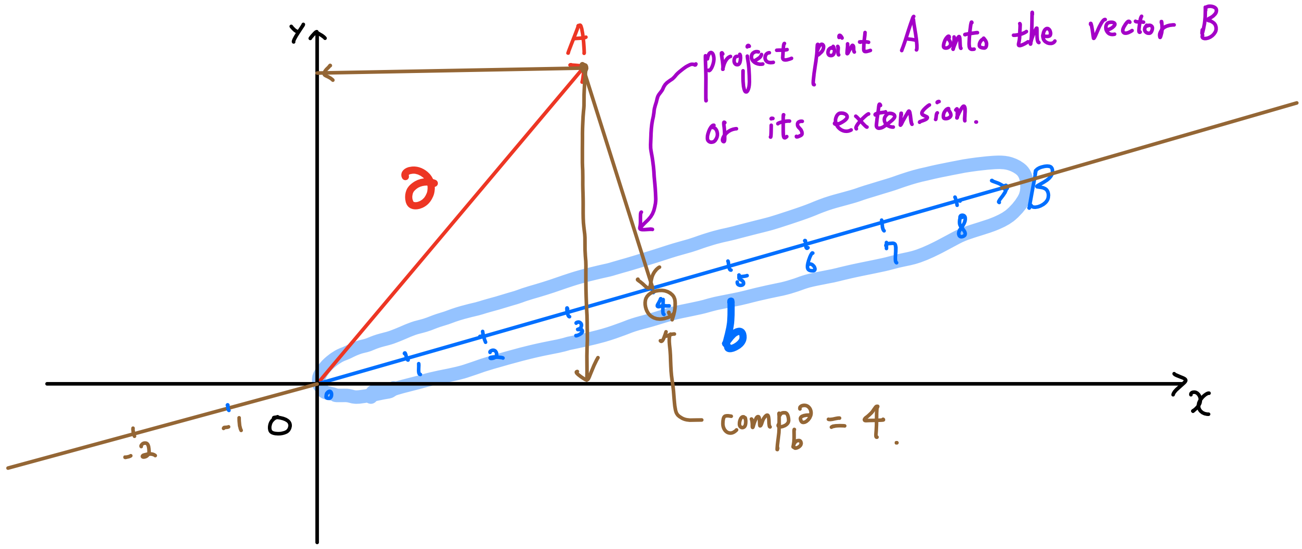
$\vec{i}, \vec{j}$ Form of a vector $\vec{a} = \langle a_1, a_2 \rangle$ is $\vec{a} = a_1 \vec{i} + a_2 \vec{j}$.

a_1 is called horizontal component of $\vec{a}$,

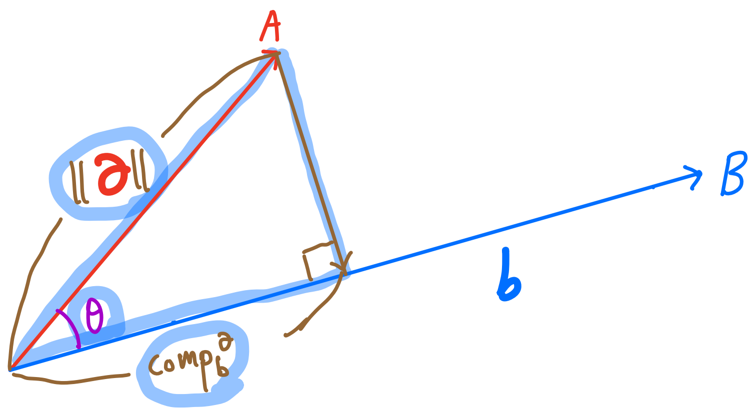
and a_2 is called vertical component of $\vec{a}$.



Given two vectors a and b,



* component of a along b, denoted by $\boxed{comp_b a}$.



$$\cos\theta = \frac{comp_b a}{\|a\|}.$$

$$\Big\downarrow \times \|a\|$$

$$\|a\|\cdot\cos\theta = comp_b a.$$

$$comp_b a = \|a\|\cdot\cos\theta$$

Recall that $\cos\theta = \dfrac{a\cdot b}{\|a\|\cdot\|b\|}$

$$comp_b a = \cancel{\|a\|}\cdot\frac{a\cdot b}{\cancel{\|a\|}\cdot\|b\|} = \frac{a\cdot b}{\|b\|}.$$

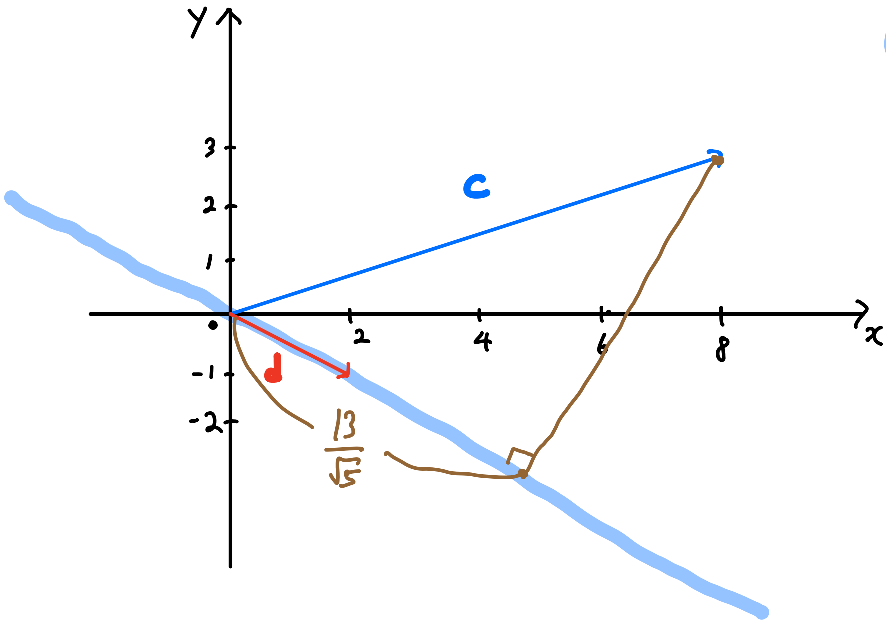
$$\Downarrow$$

Formula for $comp_b a$

If a and b are nonzero vectors, then $comp_b a = \dfrac{a\cdot b}{\|b\|}$.

Ex If $\mathbf{c} = 8i + 3j$ and $\mathbf{d} = 2i - j$, find $\mathrm{comp}_{\mathbf{d}}\mathbf{c}$.

$= \langle 8, 3 \rangle$ $\qquad$ $= \langle 2, -1 \rangle$.

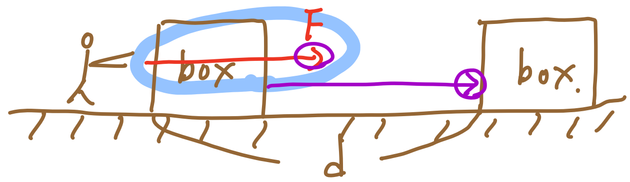


$$\mathrm{comp}_{d}c = \frac{c \cdot d}{\|d\|} = \boxed{\frac{13}{\sqrt{5}}}$$

$$c \cdot d = \langle 8, 3 \rangle \cdot \langle 2, -1 \rangle$$
$$= 8 \cdot 2 + 3 \cdot (-1)$$
$$= 16 + (-3)$$
$$= 13.$$

$$\|d\| = \|\langle 2, -1 \rangle\|$$
$$= \sqrt{2^2 + (-1)^2}$$
$$= \sqrt{4 + 1} = \sqrt{5}.$$

(Physics...!)



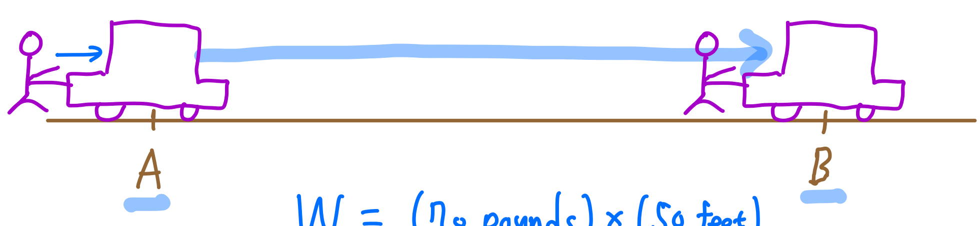
If a constant force F is applied to an object, moving it a distance d in the direction of the force, then, the "work" W done is define as follows:

$$W = (\text{the magnitude of } F) \cdot d$$

$$\boxed{\text{ft-lb}} \quad \text{lb} \cdot \text{ft}$$

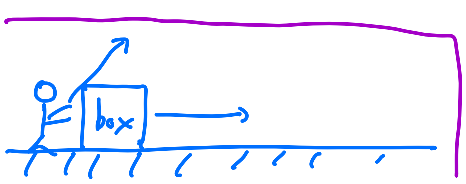
$$J = N \cdot m$$

Ex Find the work done in pushing an automobile along a level road from a point A to another point B, 50 feet from A, while exerting a constant force of 70 pounds.



$$W = (70 \text{ pounds}) \times (50 \text{ feet})$$

$$= \boxed{3{,}500 \text{ ft-lb.}}$$

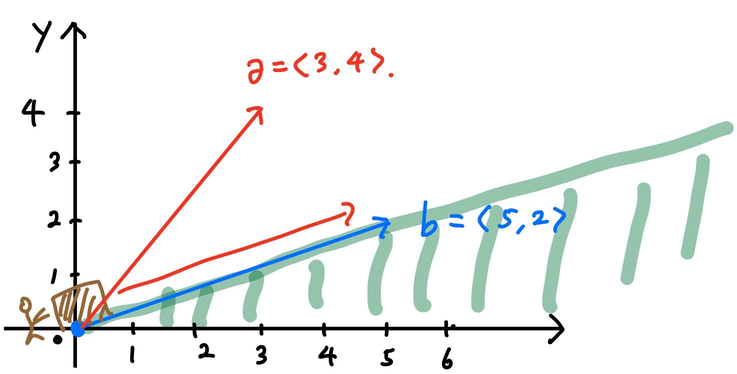


The restriction "in the direction of the force" can be deleted by using dot product!

Definition of Work

The work W done by a constant force $\mathbf{a}$ as its point of application moves along a vector $\mathbf{b}$ is $W = \mathbf{a} \cdot \mathbf{b}$.

Ex The magnitude and direction of a constant force are given by $\boxed{\mathbf{a} = 3i + 4j}$ $= \langle 3, 4 \rangle$. Find the work done if the point of application of the force moves from the origin to the point $P(5, 2)$.



$$W = \langle 3, 4 \rangle \cdot \langle 5, 2 \rangle$$
$$= 3 \cdot 5 + 4 \cdot 2$$
$$= 15 + 8$$
$$= \boxed{23}$$

Section 10.1 Infinite Sequences and Summation Notations.

- Sequence : An ordered list of numbers.

  Ex) 2, 4, 6, 8, 10 : a sequence with five terms (five numbers).

- Infinite Sequence : A sequence that consists of infinitely many numbers.

  Ex) 1, 2, 3, 4, 5, 6, 7, 8, $\cdots$: An infinite sequence of natural numbers.

* Notation for Infinite Sequence : notation 1) $a_1, a_2, a_3, \cdots, a_n, \cdots$

first number, second number, nth number

Ex) $1, 2, 3, \cdots, n, \cdots$

notation 2) $\{a_n\}$

nth number of the sequence.

Ex) $\{n\}$

nth term is n. : 1st term is 1.
2nd term is 2
3rd term is 3.
$\vdots$

Ex $\{2n+3\}$

nth term of the sequence.

: (1st number) $= 2\cdot1+3 = 2+3 = 5$

(2nd number) $= 2\cdot2+3 = 4+3 = 7$

(3rd number) $= 2\cdot3+3 = 6+3 = 9$

$\vdots$

A different point of view on Infinite sequence :

$a_1, a_2, a_3, a_4, \cdots, a_n, \cdots \longleftrightarrow$

Domain = all positive integers.

x	1	2	3	4	$\cdots$	n	$\cdots$
$f(x)$	a_1	a_2	a_3	a_4	$\cdots$	a_n	$\cdots$

(Infinite sequence can be viewed as a function whose domain is all positive integers.)

Two sequences $\begin{cases} a_1, a_2, \cdots, a_n, \cdots \\ b_1, b_2, \cdots, b_n, \cdots \end{cases}$

are equal if $a_1 = b_1,\ a_2 = b_2,\ \cdots,\ a_n = b_n,\ \cdots$.

($a_k = b_k$ for all positive integer k)

$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}, \cdots$ ← nth term

nth term

Ex 1) Find the second term of the sequence $\left\{\frac{1}{n}\right\}$. $\boxed{\frac{1}{2}}$

2) Find the second term of the sequence $\{3\}$. $\boxed{3}$

nth term

$3, 3, 3, \cdots, 3, \cdots$

nth term

$a_n = 3$ for all n.

$a_1 = 3,\ a_2 = 3,\ a_3 = 3,\ \cdots$