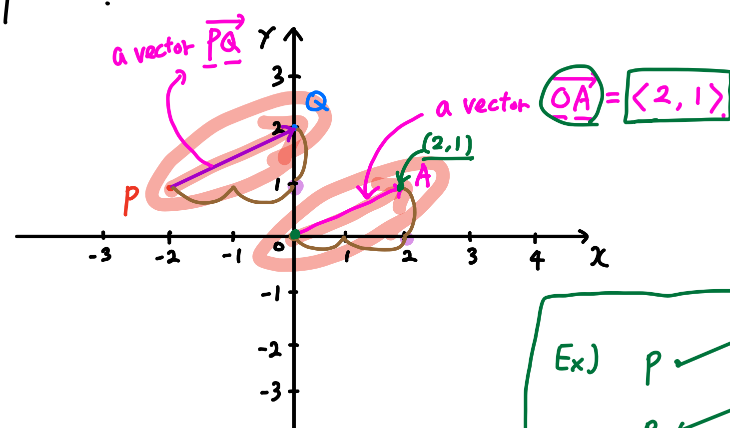
Section 8.3 continued.

- HW11: due this Friday at 11:59pm
- Please let me know if you have final schedule conflict (Ex) C1117)

In this course, we will focus on 2 dimensional vectors on xy-plane.



Ex) P → Q : $\overrightarrow{PQ}$

P ← Q : $\overrightarrow{QP}$

- $\overrightarrow{PQ}$ is a vector whose initial point is P and terminal point is Q.
- Any vector $\overrightarrow{PQ}$ can be identified with a vector $\overrightarrow{OA}$ for some point A.
- If A has a coordinate (a_1, a_2), we denote

$$\mathbf{a} = \overrightarrow{OA} = \langle a_1, a_2 \rangle$$
$(\vec{a})$

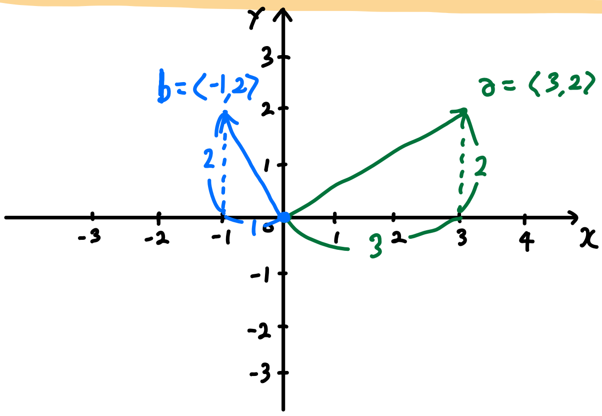
- (a_1, a_2) : a point.
- $\langle a_1, a_2 \rangle$: a vector whose initial point is $(0,0)$, and terminal point is (a_1, a_2).

- We sometime use boldface letters to denote vectors. : Ex) $\mathbf{a}, \mathbf{b}, \mathbf{u}, \mathbf{v}, \ldots$
- The magnitude of a vector is denoted by $\|\overrightarrow{OA}\|$, $\|\langle a_1, a_2 \rangle\|$, $\|\mathbf{a}\|, \ldots$

$=$ distance between $(0,0)$ and (a_1, a_2).

The "magnitude" of a vector $\mathbf{a} = \langle a_1, a_2 \rangle$, denoted by $\|\mathbf{a}\|$, is given by

$$\|\mathbf{a}\| = \|\langle a_1, a_2 \rangle\| = \sqrt{a_1^2 + a_2^2}.$$



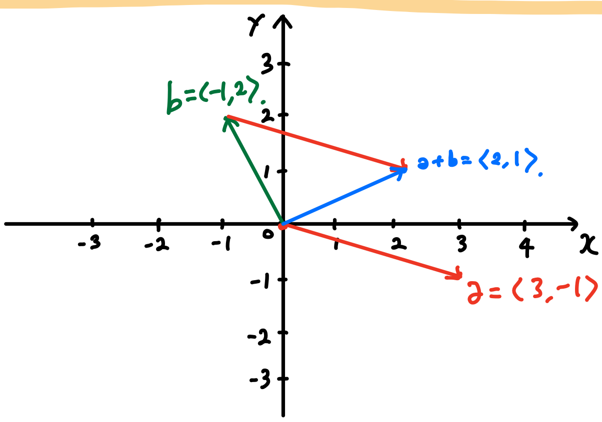
Ex $\mathbf{a} = \langle 3, 2 \rangle$. $\mathbf{b} = \langle -1, 2 \rangle$.

$\|\mathbf{a}\| = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$.

$\|\mathbf{b}\| = \sqrt{1^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$.

Definition of Addition of Vectors

$$\langle a_1, a_2 \rangle + \langle b_1, b_2 \rangle = \langle a_1 + b_1, a_2 + b_2 \rangle.$$



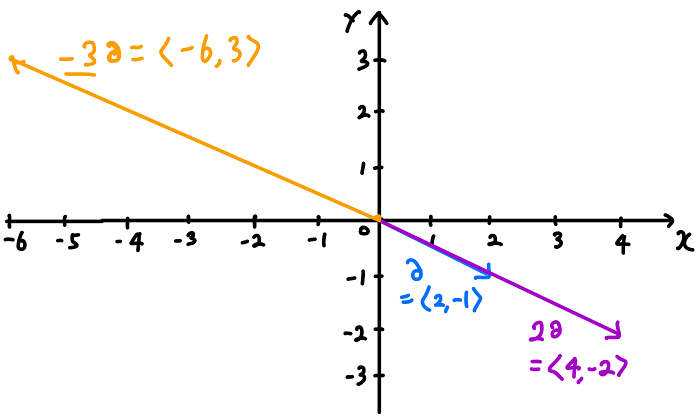
Ex $\mathbf{a} = \langle 3, -1 \rangle$. $\mathbf{b} = \langle -1, 2 \rangle$

$\mathbf{a} + \mathbf{b} = \langle 3 + (-1), -1 + 2 \rangle$

$= \boxed{\langle 2, 1 \rangle}$.

Definition of a Scalar Multiple of a vector

$$m \cdot \langle a_1, a_2 \rangle = \langle m a_1, m a_2 \rangle$$



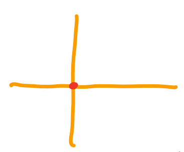
Ex $\mathbf{a} = \langle 2, -1 \rangle$.

$2 \cdot \mathbf{a} = 2 \cdot \langle 2, -1 \rangle = \langle 2 \cdot 2, 2 \cdot (-1) \rangle$
$= \langle 4, -2 \rangle$

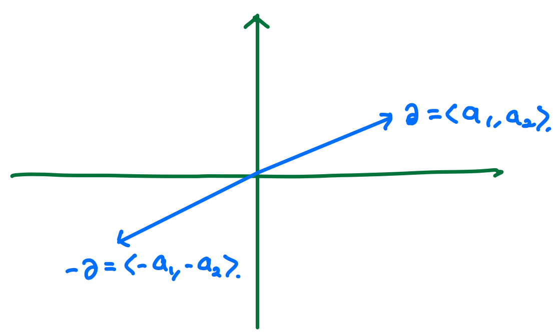
$(-3)\mathbf{a} = (-3) \cdot \langle 2, -1 \rangle = \langle (-3) \cdot 2, (-3) \cdot (-1) \rangle$
$= \langle -6, 3 \rangle$

Definition of $\mathbf{0}$ and $-\mathbf{a}$.

- $\mathbf{0} = \langle 0, 0 \rangle$



- If $\mathbf{a} = \langle a_1, a_2 \rangle$, then $-\mathbf{a} = (-1) \cdot \mathbf{a} = (-1) \cdot \langle a_1, a_2 \rangle = \langle -a_1, -a_2 \rangle$



Properties of Addition and Scalar Multiples of Vectors

(1) $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$

(2) $\mathbf{a}+(\mathbf{b}+\mathbf{c})=(\mathbf{a}+\mathbf{b})+\mathbf{c}$

(3) $\mathbf{a}+\mathbf{0}=\mathbf{a}$

(4) $\mathbf{a}+(-\mathbf{a})=\mathbf{0}$

(5) $m(\mathbf{a}+\mathbf{b})=m\mathbf{a}+m\mathbf{b}$

(6) $(m+n)\mathbf{a}=m\mathbf{a}+n\mathbf{a}$

(7) $(mn)\mathbf{a}=m(n\mathbf{a})=n(m\mathbf{a})$

(8) $1\cdot\mathbf{a}=\mathbf{a}$

(9) $0\cdot\mathbf{a}=\mathbf{0}=m\cdot\mathbf{0}$

Definition of Subtraction of Vectors

$$\langle a_1, a_2\rangle - \langle b_1, b_2\rangle = \underline{\langle a_1-b_1,\ a_2-b_2\rangle}.$$

<u>Ex</u> If $\mathbf{a}=\langle 2,-1\rangle$ and $\mathbf{b}=\langle 3,2\rangle$, find

1) $-2\mathbf{a}+3\mathbf{b}=-2\langle 2,-1\rangle+3\langle 3,2\rangle=\langle -4,2\rangle+\langle 9,6\rangle=\boxed{\langle 5,8\rangle}.$

2) $\mathbf{a}-2\mathbf{b}=\langle 2,-1\rangle-2\langle 3,2\rangle=\langle 2,-1\rangle-\langle 6,4\rangle$

$=\langle 2-6,-1-4\rangle=\boxed{\langle -4,-5\rangle}.$

* unit vector : a vector whose magnitude is 1.

<u>Ex</u> $u=\langle \frac{1}{2}, \frac{\sqrt{3}}{2}\rangle$ $\quad \|u\|=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{3}{4}}=\sqrt{1}=1.$

We give the names for two special unit vectors:

Definition of i and j:

$$i = \langle 1, 0 \rangle, \quad j = \langle 0, 1 \rangle$$

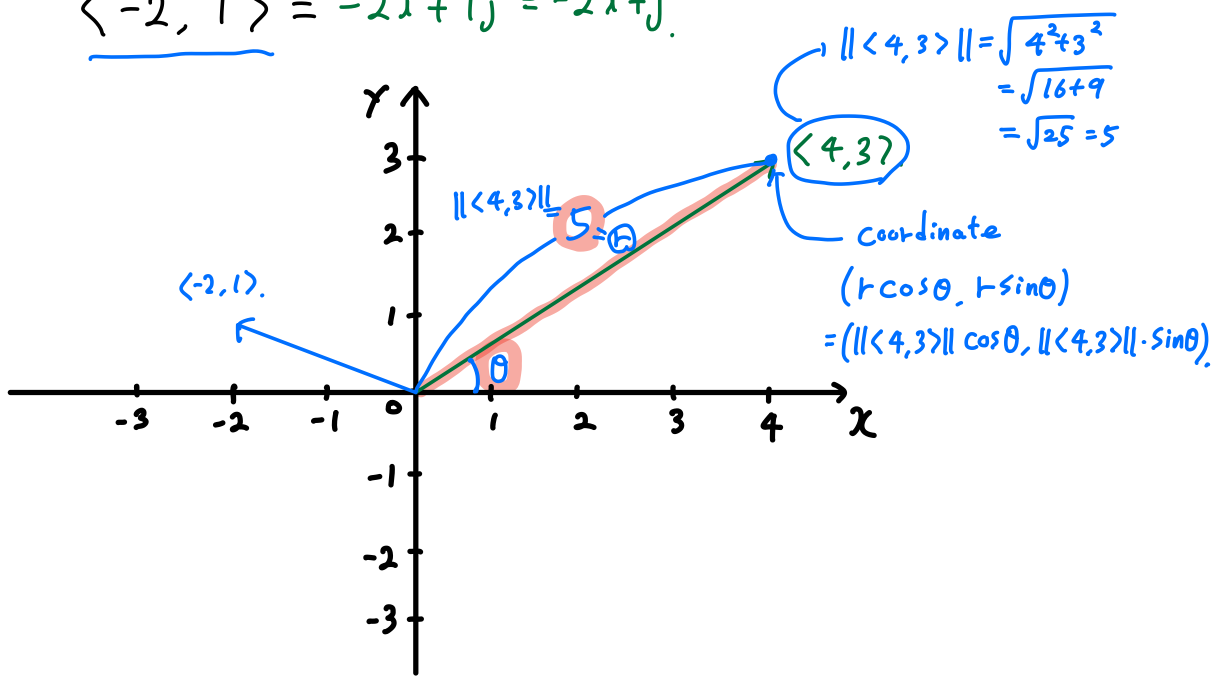
$$\langle a_1, a_2 \rangle = \langle a_1, 0 \rangle + \langle 0, a_2 \rangle = a_1 \cdot \underbrace{\langle 1, 0 \rangle}_{i} + a_2 \cdot \underbrace{\langle 0, 1 \rangle}_{j}$$

Any vector $\mathbf{a} = \langle a_1, a_2 \rangle$ can be express using i and j.

i, j Form for Vectors: $\mathbf{a} = \langle a_1, a_2 \rangle = a_1 i + a_2 j$

Ex $\langle 4, 3 \rangle = 4i + 3j$

$\langle -2, 1 \rangle = -2i + 1j = -2i + j$



i, j Form can be used when we add / subtractor vectors.

Ex If $\mathbf{a} = 4i + 2j$ and $\mathbf{b} = 3i - 5j$, then

$-\mathbf{a} + 3\mathbf{b} = -(4i+2j) + 3(3i-5j)$

$= -4i - 2j + 9i - 15j$

$= \boxed{5i - 17j} = \langle 5, -17 \rangle$.

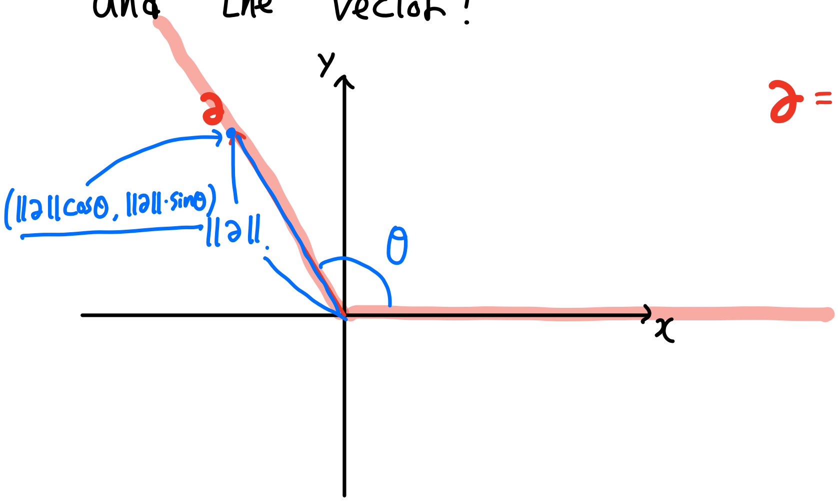
: treat i and j as if they are variables!

$(-4x - 2y + 9x - 15y = 5x - 17y.)$

The better way to represent the direction of vectors

: use the angle determined by the positive x-axis

and the vector!



$\mathbf{a} = \boxed{\langle a_1, a_2 \rangle} = \langle \|\mathbf{a}\| \cos\theta, \|\mathbf{a}\| \sin\theta \rangle$

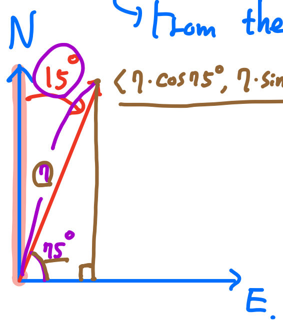
$\Downarrow$

$$\boxed{\begin{aligned} a_1 &= \|\mathbf{a}\| \cdot \cos\theta, \\ a_2 &= \|\mathbf{a}\| \cdot \sin\theta \end{aligned}}$$

Hence, we have $\underline{a_1 = \|\mathbf{a}\| \cdot \cos\theta}$ and $\underline{a_2 = \|\mathbf{a}\| \cdot \sin\theta}$.

Ex If the wind is blowing at 7 mi/hr in the direction N 15°E, express its velocity as a vector **v**.

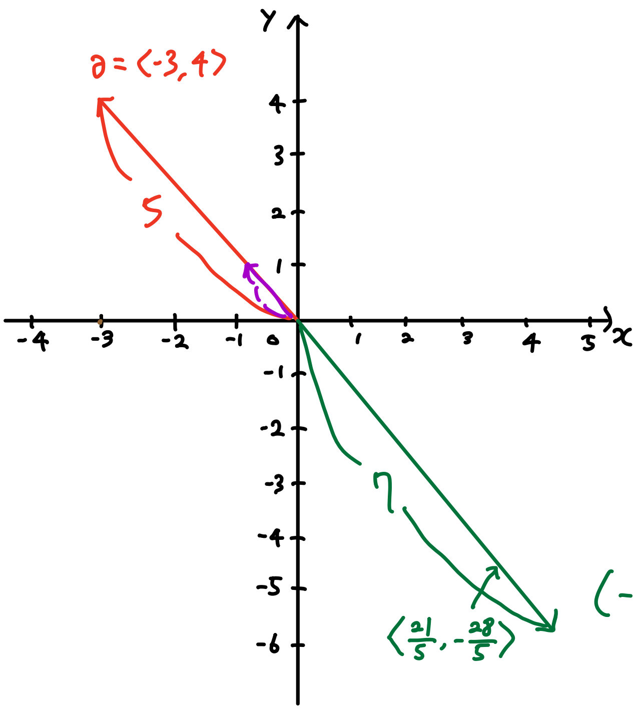
From the north to the east by 15°.



$\mathbf{v} = \langle 7\cos 75°, 7\sin 75° \rangle$

$= 7\cos 75°\, \mathbf{i} + 7\sin 75°\, \mathbf{j}.$

Ex Find a vector **b** in the opposite direction of $\mathbf{a} = \langle -3, 4 \rangle$ that has magnitude 7.



$\|\mathbf{a}\| = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5.$

First, find the unit vector that has the same direction as **a**.

Scale the **a** by $\frac{1}{5}$!

$\frac{1}{5}\cdot\mathbf{a} = \frac{1}{5}\langle -3, 4 \rangle = \langle -\frac{3}{5}, \frac{4}{5} \rangle$

unit vector and has the same direction as a

multiply (-7)

make the direction opposite

make the length 7.

$(-7)\cdot\langle -\frac{3}{5}, \frac{4}{5} \rangle = \langle (-7)\cdot(-\frac{3}{5}), (-7)\cdot\frac{4}{5} \rangle$

$= \langle \frac{21}{5}, -\frac{28}{5} \rangle$