Final : 12/13(M) 3-5pm.

HW 11: due Friday at 11:59pm.

Section 8.2. Continued.

Recall :

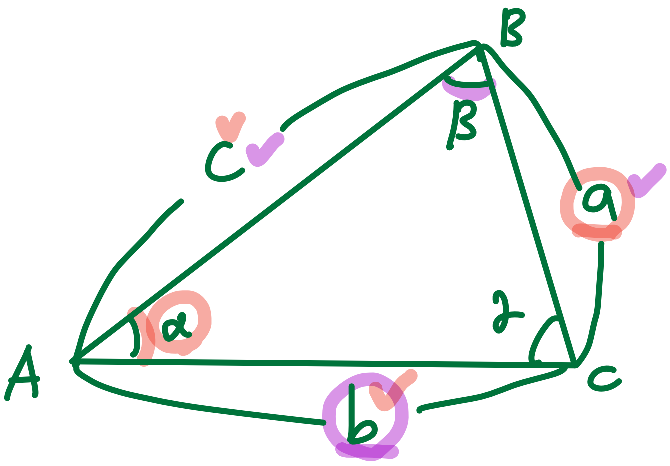
The Law of Cosines

For any triangle $\triangle ABC$, we have

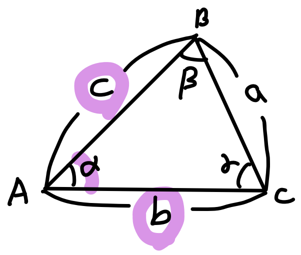
$$\underline{a^2} = \underline{b^2+c^2} - \underline{2bc}\,\underline{\cos\alpha}$$

$$\underline{b^2} = \underline{a^2+c^2} - \underline{2ac}\,\underline{\cos\beta}$$

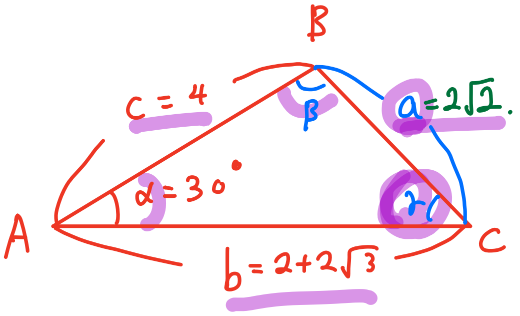
$$c^2 = a^2+b^2-2ab\cos\gamma$$



Case 4) Two sides and the angle between them are given.



Ex Solve $\triangle ABC$, given $b=2+2\sqrt{3}$, $c=4$, and $\alpha=30^\circ$.



$$a^2=b^2+c^2-2\cdot b\cdot c\cdot\cos\alpha.$$

$$=(2+2\sqrt{3})^2+4^2-2\cdot(2+2\sqrt{3})\cdot 4\cdot\boxed{\cos 30^\circ}$$

$$=2^2+(2\sqrt{3})^2+2\cdot2\cdot2\sqrt{3}+16-(4+4\sqrt{3})\cdot 4\cdot\frac{\sqrt{3}}{2}$$

$(x+y)^2=x^2+y^2+2xy$

$$=4+4\cdot3+8\sqrt{3}+16-(16+16\sqrt{3})\cdot\frac{\sqrt{3}}{2}$$

$$=4+12+8\sqrt{3}+16-(8\sqrt{3}+24)$$

$$=\underbrace{4+12}_{16}+\underbrace{\cancel{8\sqrt{3}}+16}_{32}\underbrace{-\cancel{8\sqrt{3}}-24}_{8}$$

$$=\boxed{8.}$$

$a^2=8$, $a=\sqrt{8}=\boxed{2\sqrt{2}}$

Now use $c^2=a^2+b^2-2\cdot a\cdot b\cdot\cos\gamma$ to find γ:

$$4^2=(2\sqrt{2})^2+(2+2\sqrt{3})^2-2\cdot2\sqrt{2}\cdot(2+2\sqrt{3})\cdot\cos\gamma$$

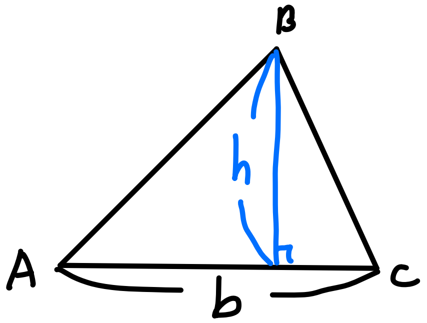
$$16=8+4+(2\sqrt{3})^2+2\cdot2\cdot2\sqrt{3}-4\sqrt{2}(2+2\sqrt{3})\cos\gamma$$

$$\cancel{16}=8+\underbrace{\cancel{4+12}}_{=16}+8\sqrt{3}-4\sqrt{2}(2+2\sqrt{3})\cos\gamma$$

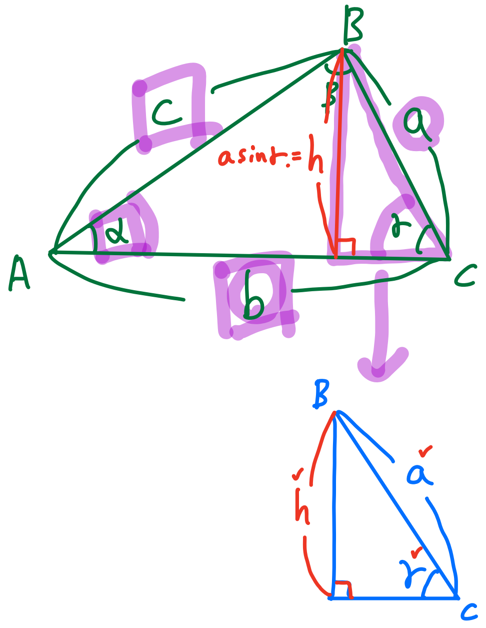
$$4\sqrt{2}(2+2\sqrt{3})\cos\gamma=8+8\sqrt{3}$$
$$=4(2+2\sqrt{3})$$

$\cos\gamma=\frac{1}{\sqrt{2}}$, $\boxed{\gamma=45^\circ}$! From $\alpha+\beta+\gamma=180^\circ$, we get $\boxed{\beta=75^\circ}$.

Recall: The area of the following triangle is



$$A = \frac{1}{2}\cdot b\cdot h$$



$$\sin\gamma = \frac{h}{a}.$$

$$\Big\downarrow \times a$$

$$\underline{a\cdot\sin\gamma = h}.$$

$$A = \frac{1}{2}\cdot b\cdot \underset{a\sin\gamma}{\underset{\shortparallel}{\underline{h}}} = \frac{1}{2}\cdot b\cdot a\sin\gamma$$

$$= \frac{1}{2}ab\sin\gamma.$$

$$= \frac{1}{2}b\cdot c\sin\alpha$$

$$= \frac{1}{2}c\cdot a\cdot\sin\beta.$$

Section 8.3 Vectors.

Scalar quantity : Quantities that are completly characterized by a single real number.

Ex height 5.7ft. / weight 170 lb.

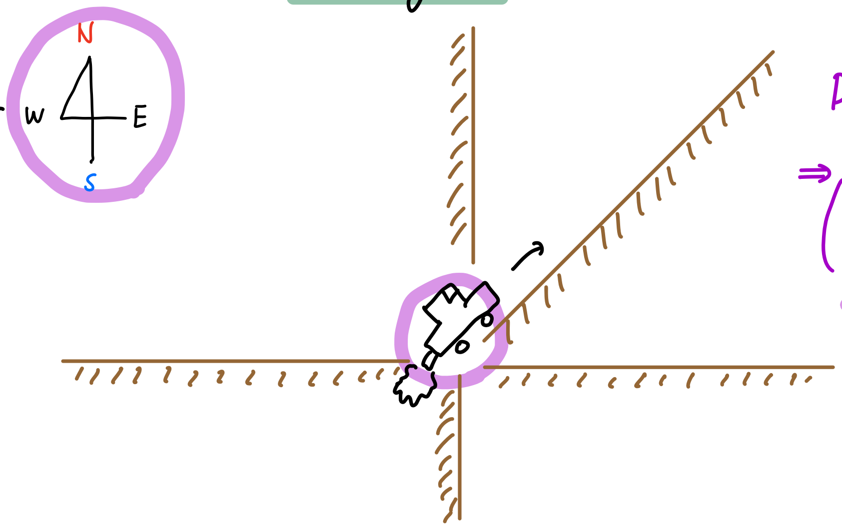
In physics, people distinguish "speed" and "velocity"

speed : "How fast the object is ?"

velocity : "How fast the object is and which direction it is heading to ?"

speed+direction.

Ex

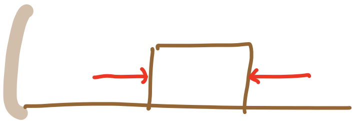


Dashbord : "55 mph".

⇒ speed - 55mph.

velocity - 55 mph to northeast

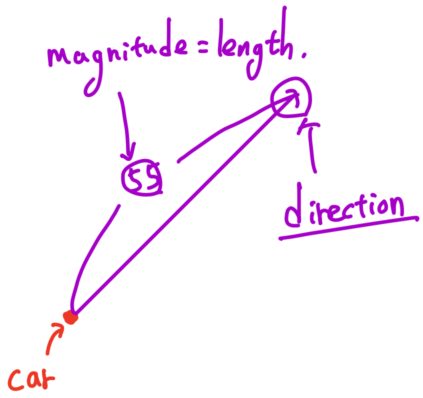
In the real word, there are many quantities that require both "magnitude" and "direction" to fully represent them.

Ex Velocity, force ( If you exert the same amount of force from different directions, the box will move differently!)

They are called "Vector quantities".

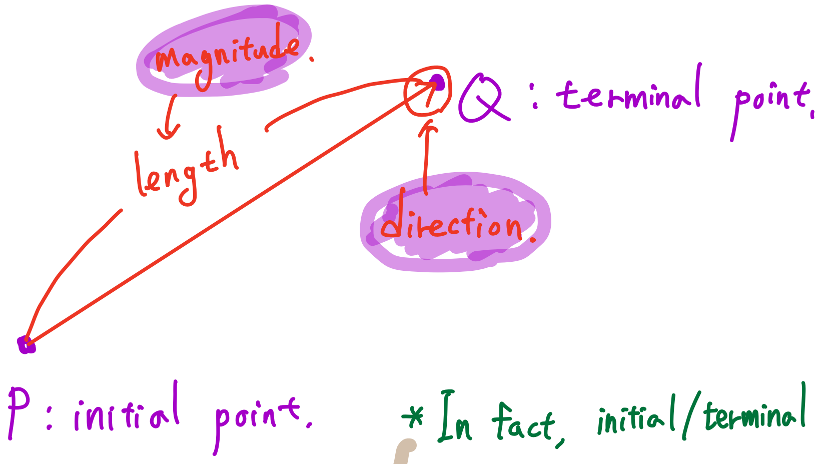
We usually represent them by using "directed line segment" (or "arrow.")

Ex In the previous example:



To determine (or represent) a vector, we need "initial point" and "terminal point".

Ex

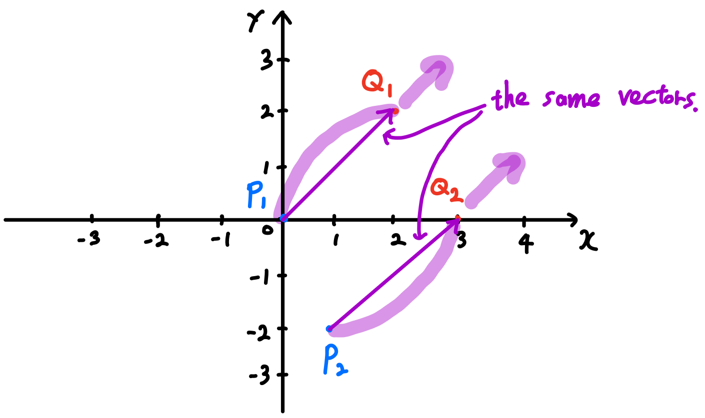


* In fact, initial/terminal points are less important than magnitude/direction

Q. When two vectors are the same?

A. Two vectors are the same if they have the same magnitude and direction.

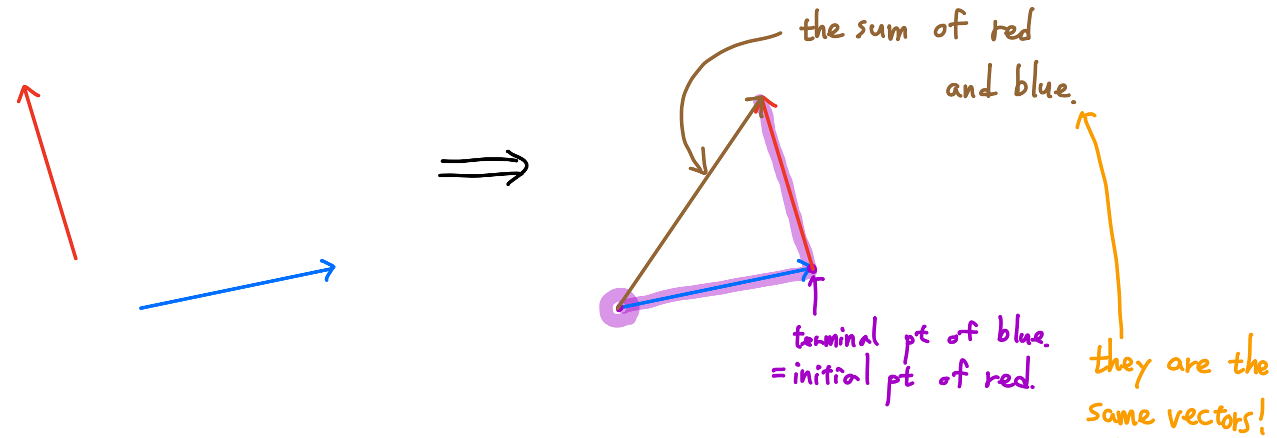
Ex



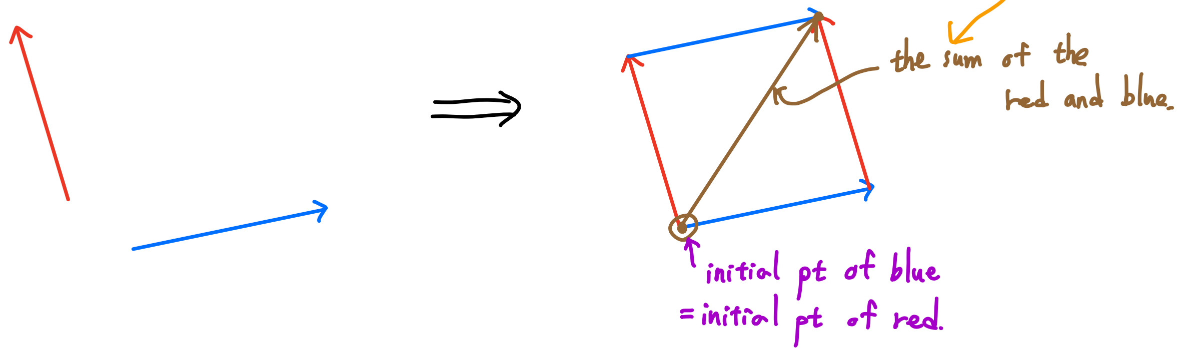
Q. Can we add vectors?

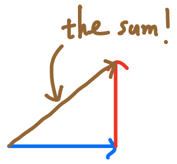
A. Yes! But the way how we add them is weird.

1st method : triangle law



2nd method : parallelogram law



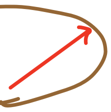
Ex The sum of  and  is 

(magnitude : 4
direction : north)

(magnitude : 4
direction : east)

* Given a vector u, $\|u\|$ is the magnitude of the vector u.

- Scalar multiplication of Vectors.

Given a vector u =  and a real number r,

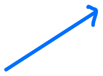
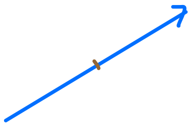
1) If $r > 0$,

$r \cdot u$ is a vector whose magnitude is $r \cdot \|u\|$ and it has the same direction as u.

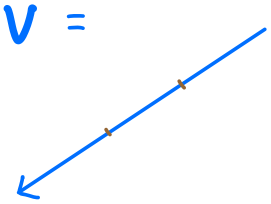
2) If $r = 0$,

$0 \cdot u$ is a zero vector.

3) $r \cdot u$ is a vector whose magnitude is $(-r) \cdot \|u\|$ and it has the opposite direction as u.

Ex V =  , $2V$ =  : magnitude - double.
the same direction.

$0V$ = · : Zero vector.

$-3V$ =  : magnitude - triple.
the opposite direction.

In this course, we will focus on 2 dimensional vectors on xy-plane.