Section 7.4 Continued

Recall

Half-Angle Identities : (1) $\sin^2 u = \frac{1-\cos 2u}{2}$

(2) $\cos^2 u = \frac{1+\cos 2u}{2}$

(3) $\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$

Hard to integrate (M212)　　easy to integrate (M212)

Ex Verify the identity $\sin^2 x \cos^2 x = \frac{1}{8}(1-\cos 4x)$ $" \frac{1-\cos 4x}{8}.$

(Recall: $\sin^2 u = \frac{1-\cos 2u}{2}$, $\cos^2 \cancel{u}_{2x} = \frac{1+\cos \cancel{2u}_{2x}}{2} \to 4x$)

$$\underbrace{\sin^2 x}_{\frac{1-\cos 2x}{2}} \underbrace{\cos^2 x}_{\frac{1+\cos 2x}{2}} = \left(\frac{1-\cos 2x}{2}\right)\cdot\left(\frac{1+\cos 2x}{2}\right) = \frac{1}{4}(1-\cos 2x)(1+\cos 2x)$$

$$= \frac{1}{4}(1-\cos^2 2x) \qquad \left(\cos^2 2x = \frac{1+\cos 4x}{2}\right)$$

$$= \frac{1}{4}\left(\frac{2}{2} - \frac{1+\cos 4x}{2}\right)$$

$$= \frac{1}{4}\cdot\frac{2-(1+\cos 4x)}{2} = \frac{2-1-\cos 4x}{8} = \boxed{\frac{1-\cos 4x}{8}}$$

Hard to integrate (M212)

Ex Express $\sin^4 t$ in terms of values of the cosine functions with exponent 1.

(Recall: $\sin^2 u = \frac{1-\cos 2u}{2}$)

$$\sin^4 t = (\sin^2 t)^2$$

$$= \left(\frac{1-\cos 2t}{2}\right)^2$$

$$= \frac{(1-\cos 2t)^2}{2^2}$$

$(x-y)^2 = x^2 - 2xy + y^2$

$$= \frac{1-2\cos 2t + \cos^2 2t}{4}$$

$$= \frac{1}{4} - \frac{\cos 2t}{2} + \frac{1}{4}\cos^2 2t$$

(Recall: $\cos^2 \cancel{u}_{2t} = \frac{1+\cos \cancel{2u}_{2t \to 4t.}}{2}$)

$$= \frac{1}{4} - \frac{\cos 2t}{2} + \frac{1}{4}\cdot\frac{1+\cos 4t}{2}$$

$$= \frac{1}{4} - \frac{\cos 2t}{2} + \frac{1}{4}\cdot\left(\frac{1}{2} + \frac{\cos 4t}{2}\right)$$

$$= \frac{1}{4} - \frac{\cos 2t}{2} + \frac{1}{8} + \frac{\cos 4t}{8}$$

$$= \frac{1}{4} + \frac{1}{8} - \frac{\cos 2t}{2} + \frac{\cos 4t}{8}$$

$$= \frac{3}{8} - \frac{\cos 2t}{2} + \frac{\cos 4t}{8}.$$

easy to integrate (M212)

Replace u by $\frac{v}{2}$

Half-Angle Identities

(1) $\sin^2 u = \frac{1-\cos 2u}{2}$

(2) $\cos^2 u = \frac{1+\cos 2u}{2}$

(3) $\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$

(1) $\sin^2 \frac{v}{2} = \frac{1-\cos v}{2}$

(2) $\cos^2 \frac{v}{2} = \frac{1+\cos v}{2}$

(3) $\tan^2 \frac{v}{2} = \frac{1-\cos v}{1+\cos v}$

take squara roots.

Half-Angle Formulas

(1) $\sin \frac{v}{2} = \pm\sqrt{\frac{1-\cos v}{2}}$

(2) $\cos \frac{v}{2} = \pm\sqrt{\frac{1+\cos v}{2}}$

(3) $\tan \frac{v}{2} = \pm\sqrt{\frac{1-\cos v}{1+\cos v}}$

Ex Find exact values for $\sin \frac{5\pi}{8}$ $\rightarrow \frac{1}{2}\cdot\frac{5\pi}{4}$

no idea...

(Recall: $\sin\frac{v}{2} = \pm\sqrt{\frac{1-\cos v}{2}}$)

replace v by $\frac{5\pi}{4}$

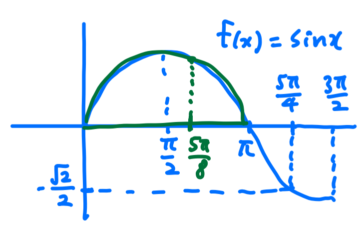
$\frac{5\pi}{8}\times 2 = \frac{5\pi}{4} = \frac{5}{4}\cdot\pi$

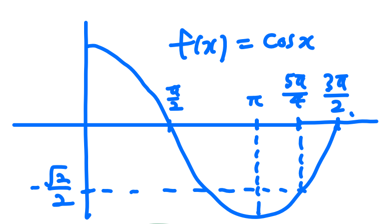
$\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

$\boxed{\cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}}$

$\sin\frac{5\pi}{8} = \pm\sqrt{\frac{1-\cos\frac{5\pi}{4}}{2}}$

$= \pm\sqrt{\frac{1-(-\frac{\sqrt{2}}{2})}{2}}$





$= \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}\cdot\frac{2}{2}} = \pm\sqrt{\frac{2+\sqrt{2}}{4}} = \pm\frac{\sqrt{2+\sqrt{2}}}{2} \Rightarrow \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$ (because $\sin\frac{5\pi}{8} > 0$)

In fact, tangent has neater forms of Half-Angle Formulas:

Half-Angle Formulas for the Tangent

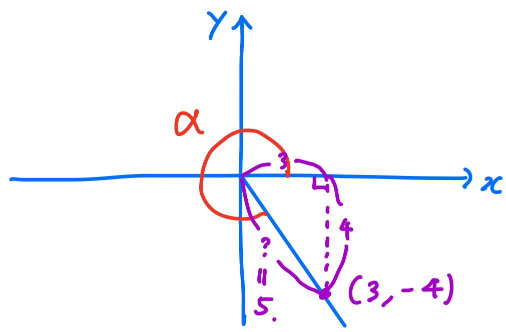
(1) $\tan\frac{v}{2} = \frac{1-\cos v}{\sin v}$

(2) $\tan\frac{v}{2} = \frac{\sin v}{1+\cos v}$

$\frac{y\text{-coordinate}}{x\text{-coordinate}} = \frac{-4}{3}$, $\Rightarrow$ choose $(3,-4)$!

Ex If $\tan\alpha = -\frac{4}{3}$ and α is in quadrant IV, find $\tan\frac{\alpha}{2}$.

$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$.



$\sin\alpha = \frac{y\text{-coordinate}}{\text{distance between } O \text{ and } (3,-4)}$

$= \frac{-4}{5}$.

$? = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$
$= 5$.

$\cos\alpha = \frac{x\text{-coordinate}}{\text{distance between } O \text{ and } (3,-4)} = \frac{3}{5}$.

$\tan\frac{\alpha}{2} = \frac{1-\frac{3}{5}}{-\frac{4}{5}} = \frac{\frac{2}{5}}{-\frac{4}{5}} \cdot \frac{5}{5} = \frac{2}{-4} = \boxed{-\frac{1}{2}}$ //

Ex Find the solutions of the equation that are in the interval $[0, 2\pi)$: $\sin 2x + \sin x = 0$.

(Recall : Double-Angle formula for sine
"$\sin 2x = \underline{2 \sin x \cos x}$")

$\sin 2x + \sin x = 0. \implies 2 \sin x \cos x + \sin x = 0.$

$\implies \sin x (2 \cos x + 1) = 0.$

$\downarrow$ Z.F.T.

$\sin x = 0$ or $2 \cos x + 1 = 0.$

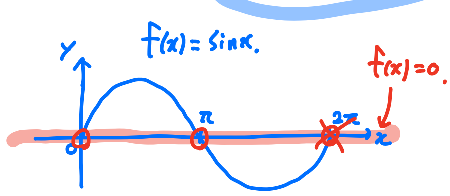
$\Downarrow$

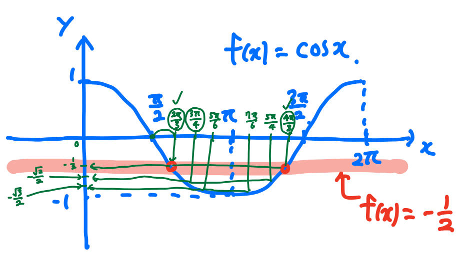
$\boxed{\sin x = 0 \quad \text{or} \quad \cos x = -\frac{1}{2}}$

$x = 0,\ x = \pi.$ $\qquad$ $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\Downarrow$

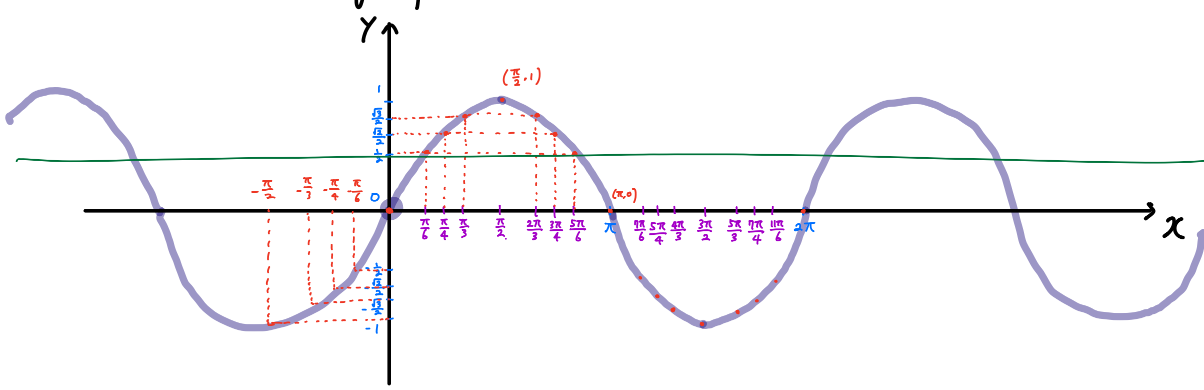
$$\boxed{x = 0,\ \pi,\ \frac{2\pi}{3},\ \frac{4\pi}{3}}$$





Section 7.6. The Inverse Trigonometric Functions.

Recall : The graph of $f(x) = \sin x$

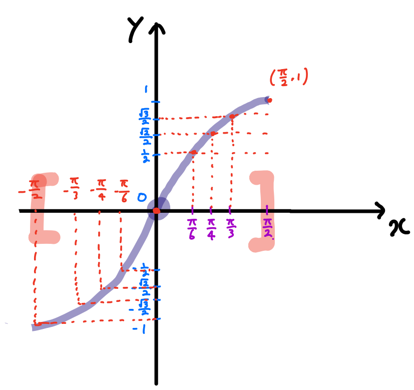


It is a graph of function, but not a graph of one-to-one function.

(vertical line test!) (horizontal line test!)

Q. How can I have a graph of one-to-one function... ?

A. Restric the domain! $(-\infty, \infty) \longrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$.



← It is one-to-one!