Section 7.3 Continued

Hw 10 is due today (11/15) at 11:59pm

Recall!

"Addition and Subtraction Formulas"

(1) $\cos(u+v) = \cos u \cos v - \sin u \sin v$

(2) $\cos(u-v) = \cos u \cos v + \sin u \sin v$

(3) $\sin(u+v) = \sin u \cos v + \cos u \sin v$

(4) $\sin(u-v) = \sin u \cos v - \cos u \sin v$

(5) $\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$

(6) $\tan(u-v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

$f(x+h) = \cos(x+h)$.

Ex If $f(x) = \cos x$ and $h \neq 0$, show that

$$\frac{f(x+h) - f(x)}{h} = \cos x\left(\frac{\cosh - 1}{h}\right) - \sin x\left(\frac{\sinh}{h}\right)$$

"difference quotient". "$(x+h)-x$".

$$\frac{f(x+h)-f(x)}{(x+h)-x}.$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\cos(x+h) - \cos x}{h}$$

* Addition formula for cos

$\cos(u+v) = \cos u \cos v - \sin u \sin v$.

$\cos(x+h) = \cos x \cosh - \sin x \sinh$

$$= \frac{\cos x \cosh - \sin x \sinh - \cos x}{h}$$

$$= \frac{\cos x \cosh - \cos x \cdot 1}{h} - \frac{\sin x \sinh}{h} = \frac{\cos x(\cosh - 1)}{h} - \frac{\sin x \sinh}{h}$$

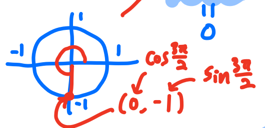
$$= \boxed{\cos x \cdot \left(\frac{\cosh - 1}{h}\right) - \sin x \cdot \left(\frac{\sinh}{h}\right)}$$

Ex Express in terms of a trigonometric function of θ alone: $\sin\left(\theta + \frac{3\pi}{2}\right) = \boxed{-\cos\theta}$.

* Recall the addition formula for sin: $\sin(u+v) = \sin u \cdot \cos v + \cos u \cdot \sin v$

$$\underset{\theta}{u},\ \underset{\frac{3\pi}{2}}{v} \qquad \underset{\theta}{u} \qquad \underset{\frac{3\pi}{2}}{v} \qquad \underset{\theta}{u} \qquad \underset{\frac{3\pi}{2}}{v}.$$

$$\sin\left(\theta + \frac{3\pi}{2}\right) = \sin\theta\ \cancel{\underbrace{\cos\frac{3\pi}{2}}_{=0}} + \cos\theta \cdot \underbrace{\sin\frac{3\pi}{2}}_{=-1} = 0 + \cos\theta \cdot (-1) = -\cos\theta.$$



Ex Express $f(x) = 2\cos 2x - 2\sqrt{3}\sin 2x$ in the form $A\cos(Bx + C)$.

Recall: $\cos(\underset{Bx}{u} + \underset{c}{v}) = \cos u \cdot \cos v - \sin u \sin v$. $\Rightarrow \cos(Bx + C) = \cos(Bx)\cos(C) - \sin(Bx)\sin(C)$

$\cos(C) = 2$, $\sin(C) = 2\sqrt{3}$ impossible!

$2^2 = 4$, $(2\sqrt{3})^2 = 12$

(sum) $= 16 \xrightarrow{\sqrt{16}} \boxed{4}$.

Factor 4! $f(x) = 2\cos 2x - 2\sqrt{3}\sin 2x$.

$$= 4 \cdot \frac{1}{2} \cdot \cos 2x - 4 \cdot \frac{\sqrt{3}}{2} \cdot \sin 2x$$

$$= 4 \cdot \left(\frac{1}{2}\cos 2x - \frac{\sqrt{3}}{2}\sin 2x\right)$$

(Find C such that $\cos(c) = \frac{1}{2}$ and $\sin(c) = \frac{\sqrt{3}}{2}$!)

↳ $c = 60°$ or $\frac{\pi}{3}$ satisfies it!

$$= 4 \cdot \left(\cos\frac{\pi}{3} \cdot \cos 2x - \sin\frac{\pi}{3} \cdot \sin 2x\right)$$

$$= 4 \cdot \cos\left(\frac{\pi}{3} + 2x\right)$$

$$= 4 \cdot \cos\left(2x + \frac{\pi}{3}\right) \Rightarrow \boxed{A = 4,\ B = 2,\ \text{and } C = \frac{\pi}{3}}$$

Section 7.4. Multiple-Angle Formulas.

Most of the formulas in this section is just special cases of the formulas from the previous section.

* Recall: Addition Formulas for sine, cosine, and tangent

$\sin(2u) = 2\sin u \cos u.$

$\sin(u+\cancel{v}^{u}) = \sin u \cos \cancel{v}^{u} + \cos u \sin \cancel{v}^{u}$

$\cos(2u) = \cos^2 u - \sin^2 u.$

$\cos(u+\cancel{v}^{u}) = \cos u \cos \cancel{v}^{u} - \sin u \sin \cancel{v}^{u}$

$$\tan(u+\cancel{v}^{u}) = \frac{\tan u + \tan \cancel{v}^{u}}{1-\tan u \tan \cancel{v}^{u}}, \quad \tan(2u) = \frac{2\tan u}{1-\tan^2 u}$$

What happen if I replace v by u ?

Double-Angle Formulas: $\sin 2u = 2\sin u \cos u.$

$$\cos 2u = \cos^2 u - \sin^2 u = 1 - 2\sin^2 u = 2\cos^2 u - 1$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u}.$$

$\sin^2 u + \cos^2 u = 1. \to \sin^2 u = (1-\cos^2 u).$
$\hookrightarrow \cos^2 u = (1-\sin^2 u).$

$\cos 2u = \cos^2 u - \sin^2 u = (1-\sin^2 u) - \sin^2 u$
$= \boxed{1-2\sin^2 u}$
$= \cos^2 u - (1-\cos^2 u) = \boxed{2\cos^2 u - 1}$

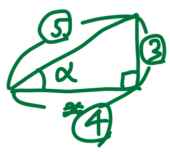
Ex If $\sin\alpha = \frac{3}{5}$ and α is an acute angle, find the exact values of $\sin 2\alpha$ and $\cos 2\alpha$.

(Recall: $\sin 2u = 2\sin u \cdot \cos u$,

$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$.)

$$\cos 2\alpha = 1 - 2\cdot\sin^2\alpha = 1 - 2\cdot\left(\frac{3}{5}\right)^2 = 1 - 2\cdot\frac{9}{25} = \frac{25}{25} - \frac{18}{25} = \boxed{\frac{7}{25}}$$

$$\sin 2\alpha = 2\cdot\sin\alpha\cdot\cos\alpha = 2\cdot\frac{3}{5}\cdot\frac{4}{5} = \boxed{\frac{24}{25}}$$



$x^2 + 3^2 = 5^2$

$x^2 + 9 = 25$, $x^2 = 16$, $x = 4$ $\Rightarrow \cos\alpha = \frac{4}{5}$

Ex Express $\sin 3\theta$ in terms of $\sin\theta$.

Recall: $\sin(u+v) = \sin u \cos v + \cos u \sin v$.

$u \to 2\theta$, $v \to \theta$

$$\sin(3\theta) = \sin(2\theta + \theta) = \sin(2\theta)\cos\theta + \cos(2\theta)\cdot\sin\theta.$$

(Recall: $\sin(2u) = 2\sin u\cos u$ ($u \to \theta$)

$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$)

$$= 2\sin\theta\cos\theta\cdot\cos\theta + (1 - 2\sin^2\theta)\cdot\sin\theta$$

$$= 2\sin\theta\cdot\cos^2\theta + (1 - 2\sin^2\theta)\cdot\sin\theta$$

($\cos^2\theta + \sin^2\theta = 1$

$\Downarrow$

$\cos^2\theta = 1 - \sin^2\theta$.)

$$= 2\sin\theta(1 - \sin^2\theta) + (1 - 2\sin^2\theta)\cdot\sin\theta$$

$$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$$

$$= \boxed{3\sin\theta - 4\sin^3\theta}.$$

Using Double-Angle Formulas, we can prove

Half-Angle Identities : (1) $\sin^2 u = \dfrac{1-\cos 2u}{2}$

(2) $\cos^2 u = \dfrac{1+\cos 2u}{2}$

(3) $\tan^2 u = \dfrac{1-\cos 2u}{1+\cos 2u}$

$\div 2$ $\qquad \div(-2)$

Recall
$$\begin{cases} \cos 2u = 2\underline{\cos^2 u} - 1 \longrightarrow 1+\cos 2u = 2\cos^2 u \\ \cos 2u = 1 - 2\sin^2 u \longrightarrow \cos 2u - 1 = -2\sin^2 u. \end{cases}$$

$$\tan^2 u = \left(\frac{\sin u}{\cos u}\right)^2 = \frac{\sin^2 u}{\cos^2 u} = \frac{\frac{1-\cos 2u}{2}}{\frac{1+\cos 2u}{2}}$$

$$= \frac{1-\cos 2u}{1+\cos 2u}$$