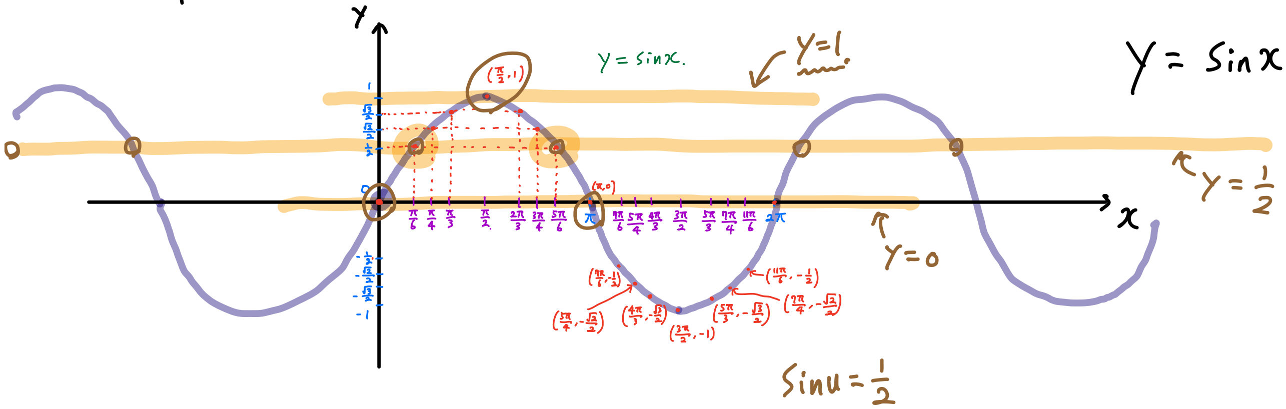
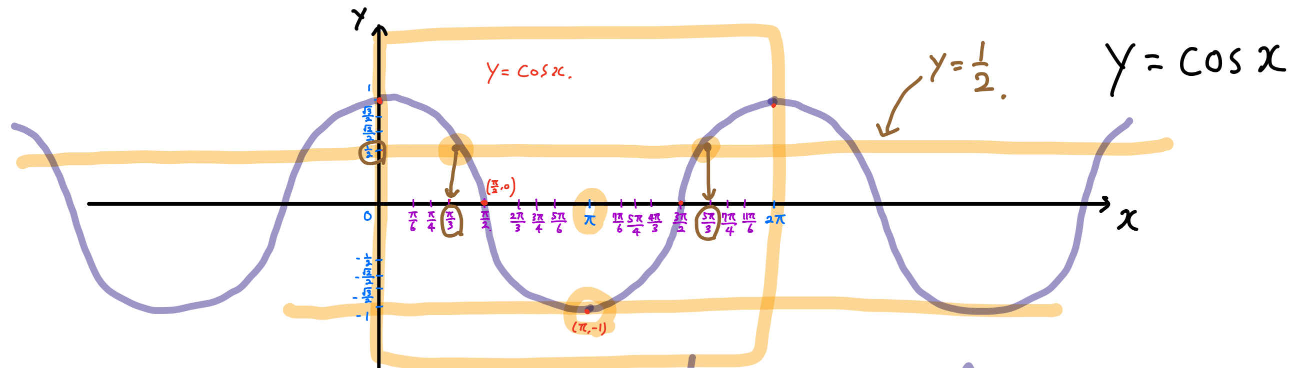
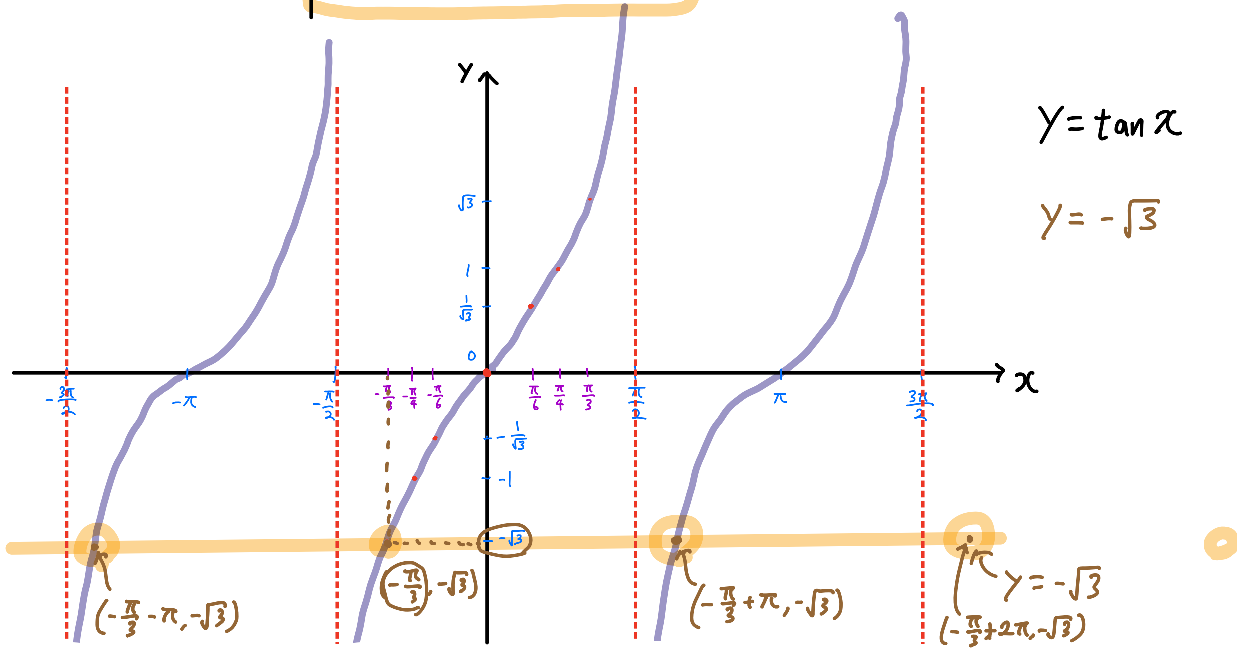
Section 7.2 Continued

Graphs of $y=\sin x$, $y=\cos x$, and $y=\tan x$.







Ex Solve the equation $\tan u = -\sqrt{3}$

From the graph, $u = -\frac{\pi}{3}$ is a solution.

$\Rightarrow u = \ldots, -\frac{\pi}{3}-2\pi, -\frac{\pi}{3}-\pi, -\frac{\pi}{3}, -\frac{\pi}{3}+\pi, -\frac{\pi}{3}+2\pi, \ldots$

$\Rightarrow \boxed{u = -\frac{\pi}{3} + n\pi \text{ for every integer } n.}$

$\vdots$

$n=0$: $-\frac{\pi}{3} + 0\cdot\pi = -\frac{\pi}{3}$

$n=1$: $-\frac{\pi}{3} + \pi$

$\vdots$

Ex (a) Solve the equation $\sin 2x = \frac{1}{2}$, and express the solutions both in radians and in degrees.

(b) Find the solutions that are in the interval $[0, 2\pi)$ $([0°, 360°))$

(a) Let $u = 2x$: $\sin u = \frac{1}{2} \Rightarrow u = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

$\Rightarrow u = \frac{\pi}{6} + 2n\pi$ and $\frac{5\pi}{6} + 2n\pi$ for every integer n.

$\Rightarrow 2x = \frac{\pi}{6} + 2n\pi$ and $\frac{5\pi}{6} + 2n\pi$ for every integer n.

$\Rightarrow \boxed{x = \frac{\pi}{12} + n\pi \text{ and } \frac{5\pi}{12} + n\pi \text{ for every integer } n}$

(b) $n = -1$: $x = \cancel{\frac{\pi}{12} - \pi}$ and $\cancel{\frac{5\pi}{12} - \pi}$ <0 <0

$n = 0$: $x = \frac{\pi}{12}$ and $\frac{5\pi}{12}$

$n = 1$: $x = \frac{\pi}{12} + \pi = \frac{13}{12}\pi$ and $\frac{5\pi}{12} + \pi = \frac{17\pi}{12}$

$\rightarrow \boxed{x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}}$

$n = 2$: $x = \cancel{\frac{\pi}{12} + 2\pi}$ and $\cancel{\frac{5\pi}{12} + 2\pi}$.

$>2\pi$ $>2\pi$

Some trigonometric equation require 'factoring'!

Ex Solve the equation $\cos x + 1 = 2\sin^2 x$

(Recall that $\sin^2 x + \cos^2 x = 1$.
$\quad -\cos^2 x \quad -\cos^2 x$
$\boxed{\sin^2 x = 1 - \cos^2 x}$)

Replace $\sin^2 x$ by $1-\cos^2 x$.

$\cos x + 1 = 2(1-\cos^2 x)$.

$\cos x + 1 = 2 - 2\cos^2 x$
$\quad -2 \quad -2$

$\cos x - 1 = -2\cos^2 x$
$+2\cos^2 x \quad +2\cos^2 x$.

$2\cos^2 x + \cos x - 1 = 0$

$2\cdot(\cos x)^2 + \cos x - 1 = 0$.

Let $X = \cos x$.

$2\cdot X^2 + X - 1 = 0$.

$(2X - 1)(X + 1) = 0$.

$\Big\downarrow$ Z.F.T.

$2X - 1 = 0$ or $X + 1 = 0$.

$X = \frac{1}{2}$ or $X = -1$.

$\cos x = \frac{1}{2}$ or $\cos x = -1$.

$\cos x = \frac{1}{2}$ or $\cos x = -1$

$x = \frac{\pi}{3} + 2n\pi, \ \frac{5\pi}{3} + 2n\pi$

$x = \pi + 2n\pi$

$\boxed{x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, \pi + 2n\pi \text{ for every integer } n.}$

Some trigonometric equation require 'fundamental identities'

Ex Find the solutions of the equation $\cos\alpha + \sin\alpha = 1$ that are in the interval $[0, 2\pi)$.

Recall: $\sin^2\alpha + \cos^2\alpha = 1$

$\cos\alpha + \sin\alpha = 1$

$\cos\alpha = 1 - \sin\alpha$

$\cos^2\alpha = (1-\sin\alpha)^2$

we take the square, so there can be extraneous solutions!

$\sin^2\alpha + \cos^2\alpha = 1.$

$\sin^2\alpha + (1-\sin\alpha)^2 = 1.$

$(x+y)^2 = x^2 + 2xy + y^2$!

$\sin^2\alpha + 1^2 - 2\cdot 1\cdot \sin\alpha + \sin^2\alpha = 1.$

$\sin^2\alpha + 1 - 2\sin\alpha + \sin^2\alpha = 1.$

$2\sin^2\alpha + 1 - 2\sin\alpha = 1.$

$2\sin^2\alpha - 2\sin\alpha = 0.$

$2\sin\alpha(\sin\alpha - 1) = 0.$

$\downarrow$ Z.F.T.

$\sin\alpha = 0$ or $\sin\alpha - 1 = 0.$

$\sin\alpha = 0$ or $\sin\alpha = 1$

$\alpha = 0, \pi$ $\qquad$ $\alpha = \frac{\pi}{2}.$

α	0	$\frac{\pi}{2}$	~~π~~
$\sin\alpha$	0	1	~~0~~
$\cos\alpha$	1	0	~~-1~~
$\sin\alpha+\cos\alpha$	1	1	~~-1~~

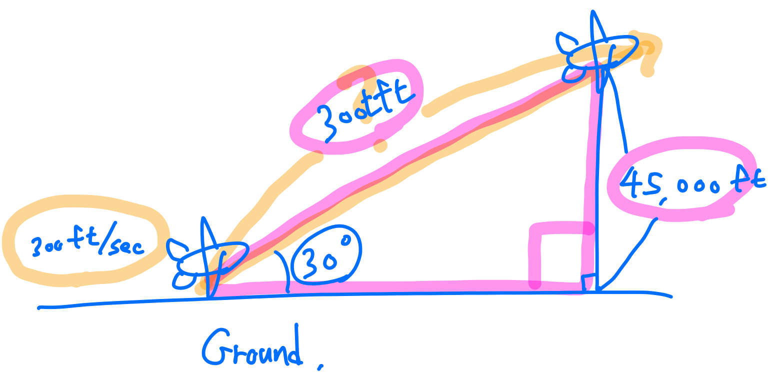
→ not a solution!

$\boxed{\alpha = 0, \frac{\pi}{2}}$

* Word Problem.

An airplane takes off at a 30° angle and travel at the rate of 300 ft/sec. How long does it take the airplane to reach an altitude of 45,000 feet?

Let t be the amount of time. (in second).



$\boxed{(\text{distance}) = (\text{speed}) \times (\text{time})}$

300t ft.   300 ft/sec   t second.

$$\frac{1}{2} = \sin 30° = \frac{45,000}{300t}$$

$$\Rightarrow \frac{1}{2} = \frac{45,000}{300t}, \quad 2 = \frac{300t}{45,000}$$

$$90,000 = 300t$$

$$\boxed{t = 300 \text{ seconds.}}$$