Section 7.1 Continued

You will see this in M212 when you integrate $f(x) = \frac{1}{\sqrt{9+x^2}}$

Ex Make the trigonometric substitution

$$x = a\tan\theta \quad \text{for} \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \quad \text{and} \quad a > 0.$$

Simplify the resulting expression : $\dfrac{1}{\sqrt{a^2+x^2}}$

replace x by $a\tan\theta$.

$$\frac{1}{\sqrt{a^2+x^2}} = \frac{1}{\sqrt{a^2+(a\tan\theta)^2}} = \frac{1}{\sqrt{a^2+a^2\tan^2\theta}}$$

$$= \frac{1}{\sqrt{a^2(1+\tan^2\theta)}}$$

$$* \quad \boxed{1+\tan^2\theta = \sec^2\theta.}$$

$$= \frac{1}{\sqrt{a^2\sec^2\theta}}$$

$$= \frac{1}{\sqrt{(a\sec\theta)^2}}$$

$$= \frac{1}{|a\sec\theta|}$$

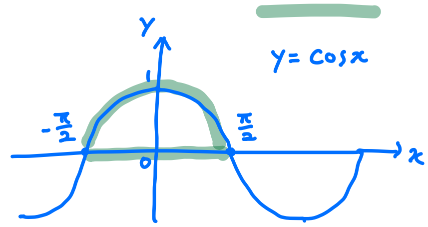
positive, positive.

$a\sec\theta$ is positive. $\Leftarrow$

$|a\sec\theta| = a\sec\theta$

$* \ \sec\theta = \dfrac{1}{\cos\theta}$

and $\cos\theta$ is $\boxed{\text{positive}}$ on the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$.



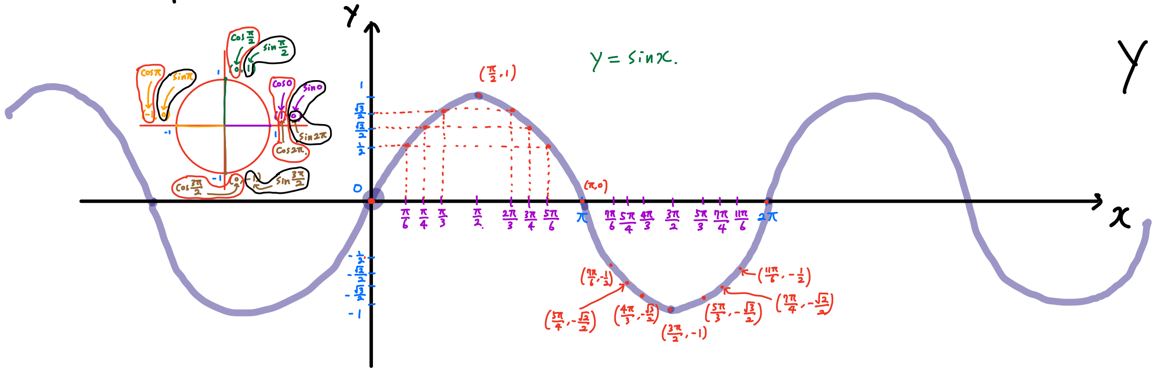
$\Rightarrow \sec\theta$ is also positive on $(-\frac{\pi}{2}, \frac{\pi}{2})$.

$$\frac{1}{a\sec\theta} = \frac{1}{a}\cdot\frac{1}{\sec\theta} = \frac{1}{a}\cdot\cos\theta = \frac{\cos\theta}{a}$$

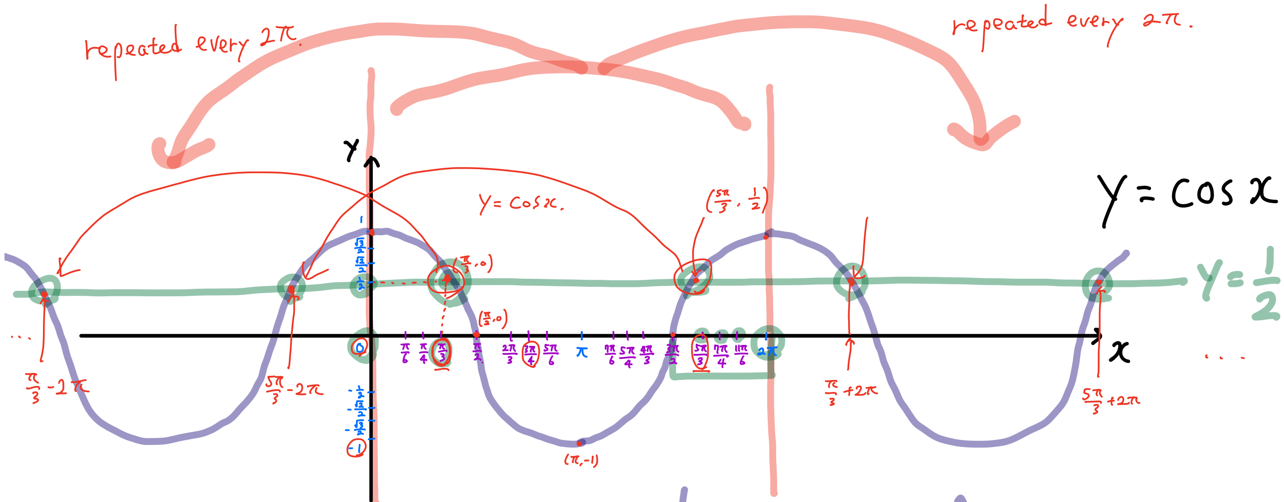
$$= \boxed{\frac{1}{a\sec\theta}} = \boxed{\frac{\cos\theta}{a}}$$

Graphs of $y=\sin x$, $y=\cos x$, and $y=\tan x$.

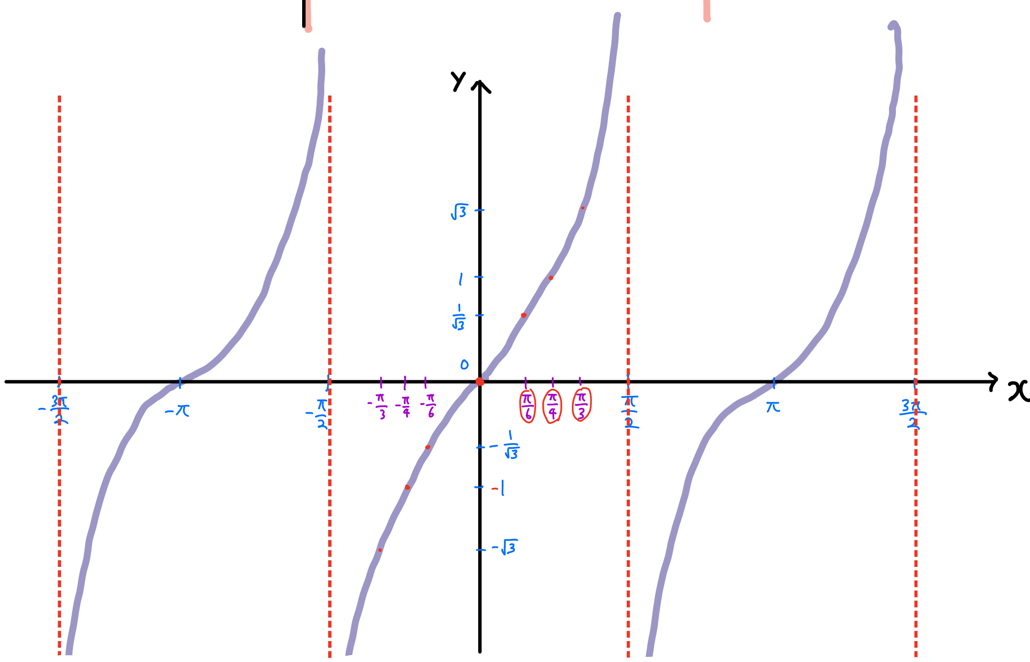
$y=\sin x$.



$y=\cos x$



$y=\tan x$



Section 7.2 Trigonometric Equations

Trigonometric equation is an equation that contains trigonometric expressions.

Ex $2\sin x + 3 = 2$

Solving trigonometric equation is to find angles that make the equation true.

Ex Solve the equation $\cos\theta = \frac{1}{2}$ if

(a) θ is in the interval $[0, 2\pi)$. $\boxed{\theta = \frac{\pi}{3} \text{ and } \theta = \frac{5\pi}{3}}$

(b) θ is any real number.

$\vdots \qquad \vdots$

$\frac{\pi}{3} - 2\pi\times 2, \quad \frac{5\pi}{3} - 2\pi\times 2,$

$\frac{\pi}{3} - 2\pi\times 1, \quad \frac{5\pi}{3} - 2\pi\times 1,$

$\theta = \frac{\pi}{3} \quad , \quad \frac{5\pi}{3}$

$\frac{\pi}{3} + 2\pi\times 1, \quad \frac{5\pi}{3} + 2\pi\times 1,$

$\frac{\pi}{3} + 2\pi\times 2, \quad \frac{5\pi}{3} + 2\pi\times 2,$

$\vdots \qquad \vdots$

instead of listing infinitely many solution, we write the answer as follows:

$\boxed{\theta = \frac{\pi}{3} + 2\pi n, \quad \frac{5\pi}{3} + 2\pi n \quad \text{for every integer } n.}$

Proof) Draw the graph of $y = \cos x$ and $y = \frac{1}{2}$ on the same coordinate plane. Then, observe the intersections.