Section 7.1 Verifying Trigonometric Identities.

"Trigonometric expression" is an expression that contains trigonometric functions.

Ex $2 \cdot \sin x - 1$, $\cos x \cdot \cot x - \sin x$

"Trigonometric identity" is an identity that contains trigonometric expressions.

Ex $\sin^2 x + \cos^2 x = 1$, $\sin x + \cos x \cdot \cot x = \csc x$?

Q : How we verify the trigonometric identities ... ?

A : Use the definitions and the fundamental identities.

$$\begin{cases} \tan\theta = \dfrac{\sin\theta}{\cos\theta} \\ \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta} \\ \csc\theta = \dfrac{1}{\sin\theta} \\ \sec\theta = \dfrac{1}{\cos\theta} \end{cases} \qquad \begin{cases} \sin^2\theta + \cos^2\theta = 1 \\ 1 + \tan^2\theta = \sec^2\theta \\ 1 + \cot^2\theta = \csc^2\theta. \end{cases}$$

Ex Verify the idenity $\underbrace{\sin x + \cos x \cot x}_{L.H.S.} = \underbrace{\csc x}_{R.H.S.}$

the same

$(R.H.S.) = \csc x = \dfrac{1}{\sin x}$

$(L.H.S) = \sin x + \cos x \cot x = \sin x + \cos x \cdot \dfrac{\cos x}{\sin x}$

$= \dfrac{\sin^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x} = \dfrac{\overset{1}{\sin^2 x + \cos^2 x}}{\sin x} = \dfrac{1}{\sin x}$

Ex Verify the idenity $\underbrace{(\sec u - \tan u)(\csc u + 1)}_{L.H.S.} = \underbrace{\cot u}_{R.H.S.}$

$(R.H.S.) = \cot u = \boxed{\dfrac{\cos u}{\sin u}}$

$(L.H.S.) = (\sec u - \tan u)(\csc u + 1)$

$= \left(\dfrac{1}{\cos u} - \dfrac{\sin u}{\cos u}\right)\left(\dfrac{1}{\sin u} + \dfrac{\sin u}{\sin u}\right)$

$= \left(\dfrac{1-\sin u}{\cos u}\right) \cdot \left(\dfrac{1+\sin u}{\sin u}\right) = \dfrac{(1-\sin u)(1+\sin u)}{\cos u \cdot \sin u}$

$= \dfrac{1-\sin^2 u}{\cos u \cdot \sin u}$

$= \dfrac{\cos^2 u}{\cos u \cdot \sin u} = \boxed{\dfrac{\cos u}{\sin u}}$

the same

$(x-y)(x+y) = x^2 - y^2$

$\sin^2 u + \cos^2 u = 1$
$-\sin^2 u \quad -\sin^2 u$
$\cos^2 u = 1 - \sin^2 u$

Ex Verify the identity $\dfrac{\tan^2 x}{\sec x + 1} = \dfrac{1 - \cos x}{\cos x}$

Proof 1) $(RHS) = \dfrac{1-\cos x}{\cos x}$

$(LHS) = \dfrac{\tan^2 x}{\sec x + 1} = \dfrac{\left(\frac{\sin x}{\cos x}\right)^2}{\frac{1}{\cos x} + 1} = \dfrac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos x} + \frac{\cos x}{\cos x}}$

$= \dfrac{\frac{\sin^2 x}{\cos^2 x}}{\frac{(1+\cos x)}{\cos x}}$ $\qquad \dfrac{\frac{a}{b}}{\frac{c}{d}} = \dfrac{ad}{bc}$

$= \dfrac{\sin^2 x \cdot \cos x}{\cos^2 x \cdot (1+\cos x)} = \dfrac{\sin^2 x}{\cos x \cdot (1+\cos x)}$

$= \dfrac{1^2 - \cos^2 x}{\cos x \cdot (1+\cos x)}$

$= \dfrac{\cancel{(1+\cos x)}(1-\cos x)}{\cos x\,\cancel{(1+\cos x)}}$

$= \dfrac{1-\cos x}{\cos x}$

the same

$\sin^2 x + \cancel{\cos^2 x} = 1$
$-\cancel{\cos^2 x} \quad -\cos^2 x$
$\sin^2 x = 1 - \cos^2 x$

Proof 2

$1 + \tan^2 x = \sec^2 x.$

$\tan^2 x = \sec^2 x - 1.$

$(L.H.S.) = \dfrac{\tan^2 x}{\sec x + 1} = \dfrac{\sec^2 x - 1^2}{\sec x + 1} = \dfrac{\cancel{(\sec x + 1)}(\sec x - 1)}{\cancel{\sec x + 1}}$

$= \sec x - 1$

$= \dfrac{1}{\cos x} - \dfrac{\cos x}{\cos x}$

$= \dfrac{1-\cos x}{\cos x} = (R.H.S.)$

Ex Verify the identity $\underbrace{(\sec t + \tan t)^2}_{L.H.S.} = \underbrace{\dfrac{1+\sin t}{1-\sin t}}_{R.H.S.}$

$(L.H.S.) = (\sec t + \tan t)^2$

$= \left(\dfrac{1}{\cos t} + \dfrac{\sin t}{\cos t}\right)^2$

$= \left(\dfrac{1+\sin t}{\cos t}\right)^2$ $\qquad \left(\sin^2 t + \cos^2 t = 1,\ \cos^2 t = 1 - \sin^2 t\right)$

$= \dfrac{(1+\sin t)^2}{\cos^2 t}$

$= \dfrac{(1+\sin t)^2}{1^2 - \sin^2 t} = \dfrac{(1+\sin t)^2}{(1+\sin t)(1-\sin t)} = \dfrac{1+\sin t}{1-\sin t} = (R.H.S.)$

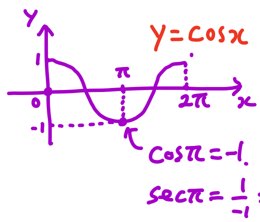
Ex Show that $\boxed{\sec t = \sqrt{\tan^2 t + 1}}$ is not an identity.

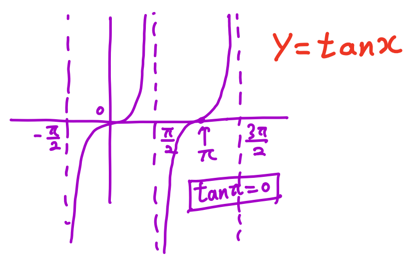
find a value t such that the given equation is not true!

Observation $\sec t = \frac{1}{\cos t}$ can be negative,

but $\sqrt{\tan^2 t + 1}$ is always non-negative.

$\Rightarrow$ Find t such that $\sec t = \frac{1}{\cos t}$ is negative!





If we replace t by π, $(LHS) = \sec\pi = \frac{1}{\cos\pi} = \frac{1}{-1} = -1$, different!

and $(RHS) = \sqrt{\tan^2\pi + 1} = \sqrt{0^2+1} = \sqrt{1} = 1$

Thus, $\sec t = \sqrt{\tan^2 t + 1}$ is not an identity.