(Section 6.6 Continued)

Office Hour: 11/4, 10:30 am - 11:30 am / HW 9 : due next Monday (11/8) at 11:59 pm

Theorem on the graph of $y = a\tan(bx+c)$

If $y = a\tan(bx+c)$ for nonzero real numbers a, b and c, then

(1) the period is $\frac{\pi}{|b|}$.

(2) successive vertical asymptotes for the graph of one branch may be found by solving the inequality

$$\boxed{-\frac{\pi}{2} < bx + c < \frac{\pi}{2}}$$

* We cannot argue about the 'amplitude'.

the height from the center line to peak.

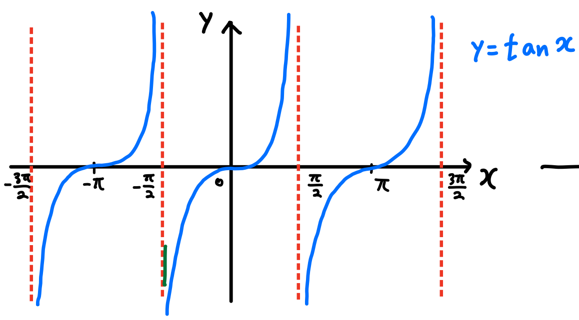
Ex Sketch the graph of $y = \underbrace{\frac{1}{3}}_{a} \tan(\underbrace{2}_{b}x - \underbrace{\frac{\pi}{2}}_{c})$

From the above theorem, the period is $\frac{\pi}{|b|} = \frac{\pi}{|2|} = \boxed{\frac{\pi}{2}}$.

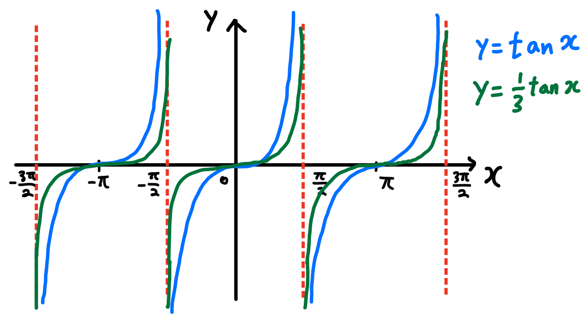
Also, $\boxed{-\frac{\pi}{2} < 2x - \frac{\pi}{2} < \frac{\pi}{2}} \Rightarrow 0 < 2x < \pi$, $\boxed{0 < x < \frac{\pi}{2}}$

$y = \tan x \to y = \frac{1}{3}\tan x \to y = \frac{1}{3}\tan 2x \to y = \frac{1}{3}\tan(2x - \frac{\pi}{2})$,

Hence one branch of the tangent function is on $(0, \frac{\pi}{2})$

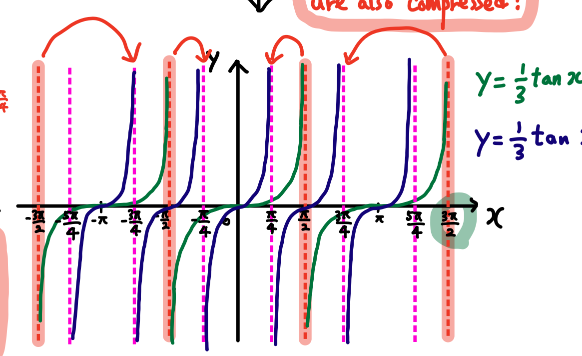


$\longrightarrow$



$\downarrow$

* Vertical Asymptotes are also compressed!



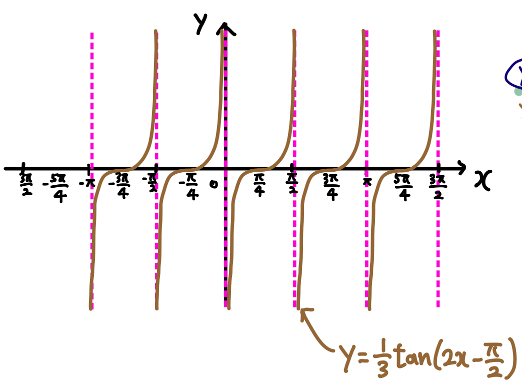
$y = \frac{1}{3}\tan 2x$ ($x - \frac{\pi}{4}$)

$y = \frac{1}{3}\tan(2x - \frac{\pi}{2})$

$= \frac{1}{3}\tan(2(x - \frac{\pi}{4}))$

$\longleftarrow$

translate the graph $\frac{\pi}{4}$ units $\longrightarrow$

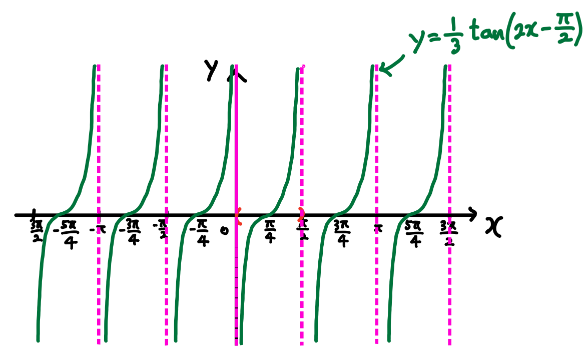


* In fact, you can draw the graph directly using the theorem.

$y = \frac{1}{3}\tan(\underbrace{2}_{b}x - \frac{\pi}{2})$: period $\Rightarrow \frac{\pi}{|b|} = \boxed{\frac{\pi}{2}}$

$-\frac{\pi}{2} < 2x - \frac{\pi}{2} < \frac{\pi}{2} \Rightarrow 0 < x < \frac{\pi}{2}$

$(0, \frac{\pi}{2})$



Theorem on the graph of $y = a\cot(bx+c)$

If $y = a\cot(bx+c)$ for nonzero real numbers a, b and c, then

(1) the period is $\dfrac{\pi}{|b|}$.

(2) successive vertical asymptotes for the graph of one branch may be found by solving the inequality

$$0 < bx+c < \pi$$

Ex Sketch the graph of

$$\begin{cases} y = 2\cot(3x-\pi) \\ y = 2\cot(-3x+\pi) \\ y = -2\cot(3x-\pi) \\ y = -2\cot(-3x+\pi) \end{cases}$$

$y = 2\cot(3x-\pi)$: Period : $\frac{\pi}{3}$

$0 < 3x-\pi < \pi$
$\pi < 3x < 2\pi \rightarrow \frac{\pi}{3} < x < \frac{2\pi}{3} \Rightarrow \left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$

$y = 2\cot(-3x+\pi)$: Period : $\frac{\pi}{|-3|} = \frac{\pi}{3}$

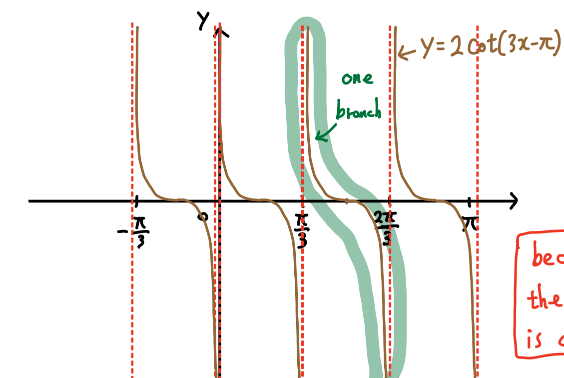
$0 < -3x+\pi < \pi$
$-\pi < -3x < 0 \rightarrow \frac{\pi}{3} > x > 0 \Rightarrow \left(0, \frac{\pi}{3}\right)$

$y = -2\cot(3x-\pi)$: Period : $\frac{\pi}{3}$

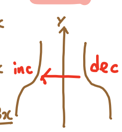
$0 < 3x-\pi < \pi$
$\pi < 3x < 2\pi \rightarrow \frac{\pi}{3} < x < \frac{2\pi}{3} \Rightarrow \left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$

$y = -2\cot(-3x+\pi)$: Period : $\frac{\pi}{|-3|} = \frac{\pi}{3}$

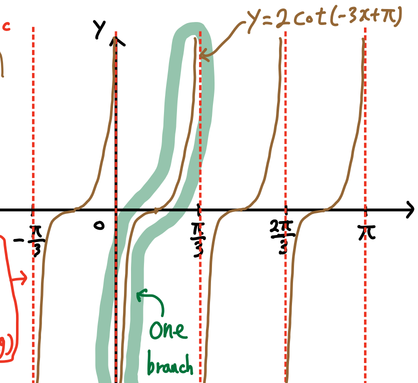
$0 < -3x+\pi < \pi$
$-\pi < -3x < 0 \rightarrow \frac{\pi}{3} > x > 0 \Rightarrow \left(0, \frac{\pi}{3}\right)$

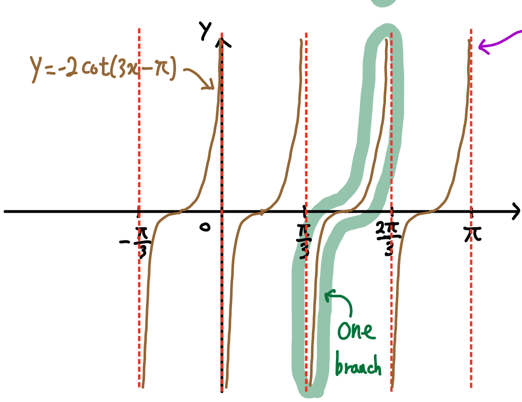


$y = \cot x$
$\downarrow$
$y = 2\cot x$ inc
$\downarrow$
$y = 2\cot 3x$
$\downarrow$ reflection through y-axis
$y = 2\cot(-3x)$
$\downarrow$
$y = 2\cot(-3x+\pi)$



because of this reflection, the shape of the branch is changed. (decreasing $\rightarrow$ increasing)

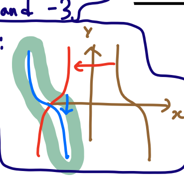


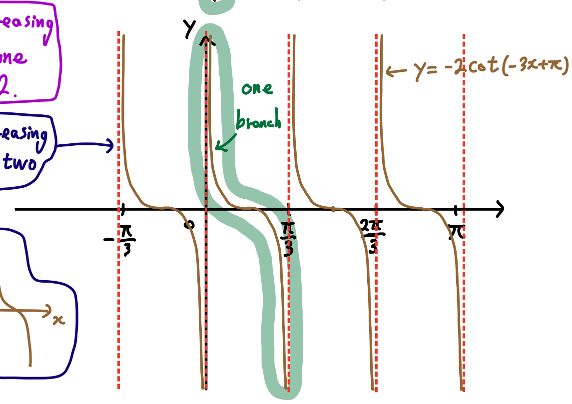


The branch is increasing because we have one reflection due to -2.

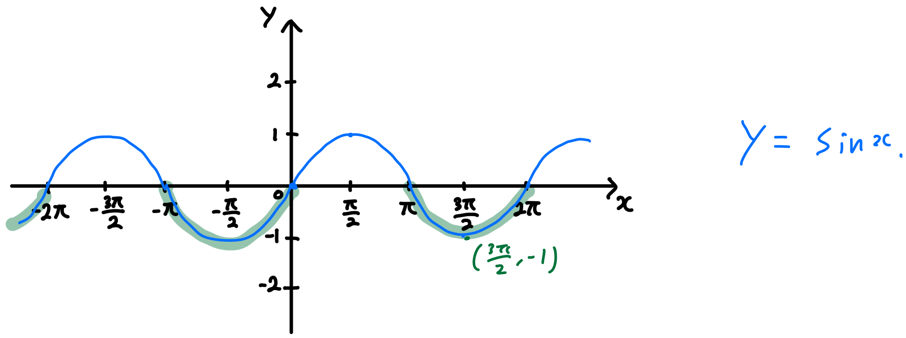
The branch is decreasing because we have two reflections due to -2 and -3.

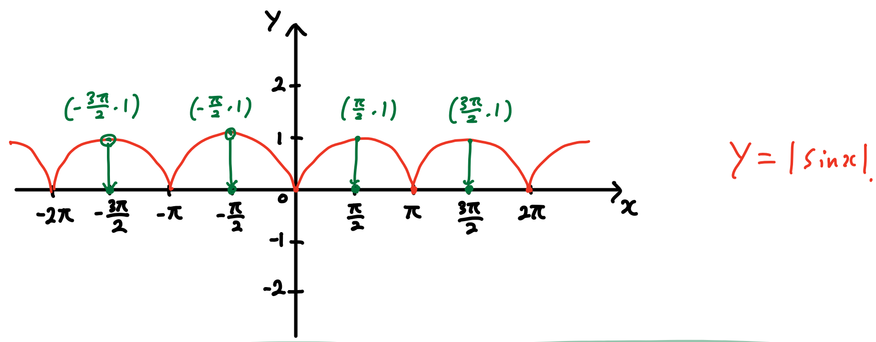
See :

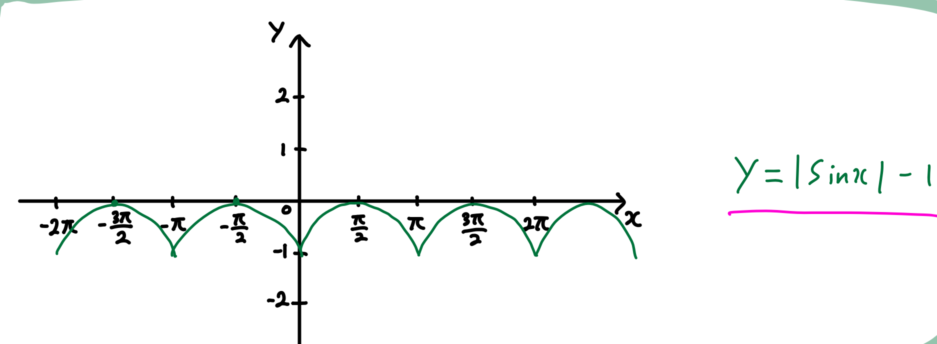




Ex Sketch the graph of $y = |\sin x| - 1$.



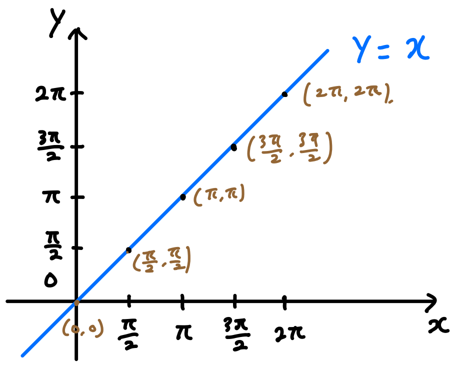


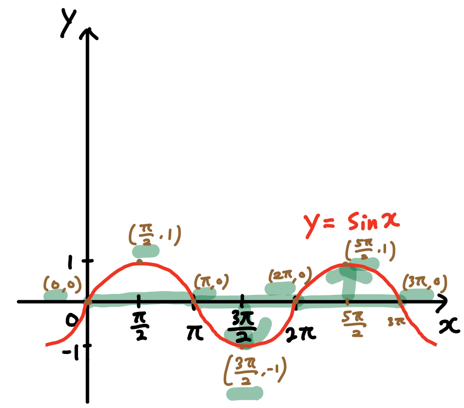


Ex Sketch the graph of $y = x + \sin x$.

$y = x$

$y = \sin x$

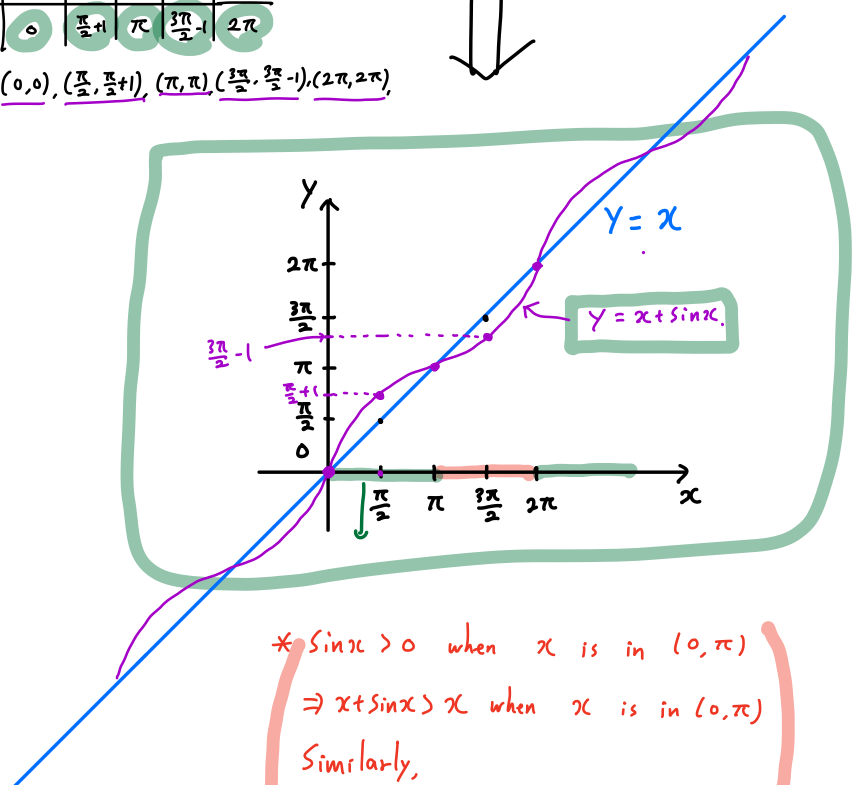




x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y=x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y=\sin x$	0	1	0	-1	0
$y=x+\sin x$	0	$\frac{\pi}{2}+1$	π	$\frac{3\pi}{2}-1$	2π

$(0,0)$, $(\frac{\pi}{2}, \frac{\pi}{2}+1)$, (π,π), $(\frac{3\pi}{2}, \frac{3\pi}{2}-1)$, $(2\pi, 2\pi)$

$\Downarrow$



* $\sin x > 0$ when x is in $(0, \pi)$

$\Rightarrow x + \sin x > x$ when x is in $(0, \pi)$

Similarly,

$\sin x < 0$ when x is in $(\pi, 2\pi)$

$\Rightarrow x + \sin x < x$ when x is in $(\pi, 2\pi)$