(Section 6.5 Continued)

Finding the equation from the graph.

Ex Express the equation for the cosine wave shown below in the form $y = \overset{6}{a}\cos(\overset{\frac{\pi}{2}}{b}x + c)$ for $a > 0$, $b > 0$, and the least positive real number c.

amplitude: $|a| = 6$ → $\boxed{a = 6}$

period: $\frac{2\pi}{|b|} = 4$

$2\pi = 4 \cdot |b|$

$2\pi = 4 \cdot b$

$\boxed{b = \frac{2\pi}{4} = \frac{\pi}{2}}$

$\boxed{y = 6 \cdot \cos\left(\frac{\pi}{2}x\right)}$

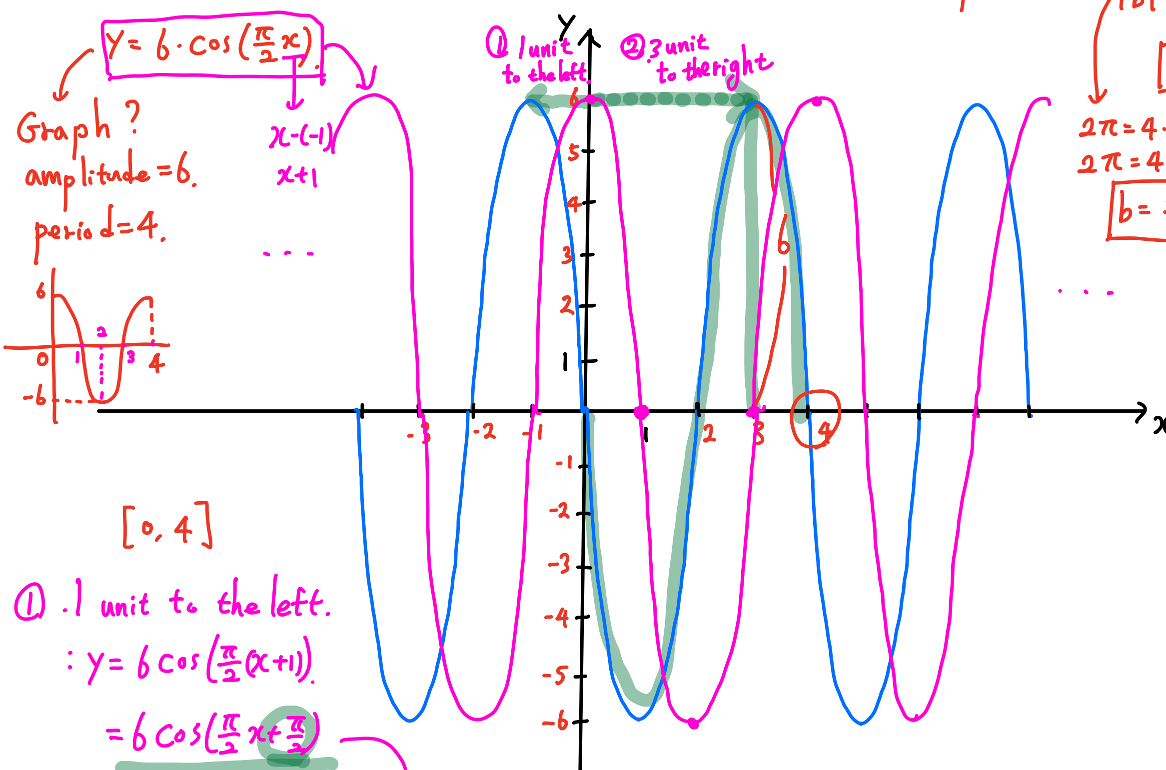
Graph ?

amplitude = 6.

period = 4.

$x - (-1)$

$x + 1$



[0, 4]

① 1 unit to the left.

: $y = 6\cos\left(\frac{\pi}{2}(x+1)\right)$.

$= 6\cos\left(\frac{\pi}{2}x + \frac{\pi}{2}\right)$

$\boxed{a = 6,\ b = \frac{\pi}{2},\ c = \frac{\pi}{2}}$

② 3 units to the right

: $y = 6\cos\left(\frac{\pi}{2}(x-3)\right)$.

$= 6\cos\left(\frac{\pi}{2}x - \frac{3\pi}{2}\right)$

Section 6.6 Additional Trigonometric Graphs

(tangent, cotangent, ~~secant~~, and ~~cosecant~~)

In this section, we will deal with the trigonometric functions of the forms ① $\boxed{y = a\tan(bx+c)}$

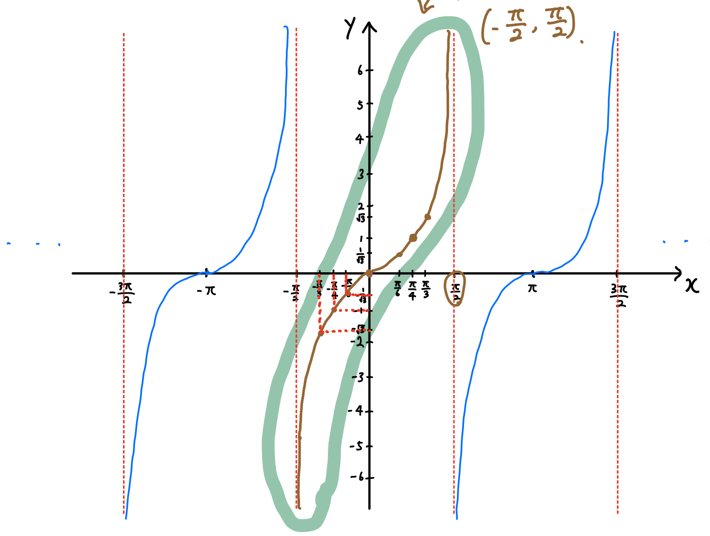
② $\boxed{y = a\cot(bx+c)}$

The idea is the same as Section 6.5!

We first recall the graph of $y=\tan x$ and derive the graph of $y = \cot x = \frac{1}{\tan x}$.

"a branch" of the tangent function $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

odd function!



Recall: $y=\sin x$ is odd

$y=\cos x$ is even.

$\sin(-x) = -\sin x$.

$\cos(-x) = \cos x$.

$\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\tan x$

$y = \tan x$

✓ Domain : $\mathbb{R} - \left\{\dots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots\right\}$ $\quad \frac{(\text{odd})\cdot\pi}{2}$

✓ Period : π

Some values of $\tan x$.

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y=\tan x$	undefined	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

$\boxed{\tan x = \frac{\sin x}{\cos x}}$

$\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$

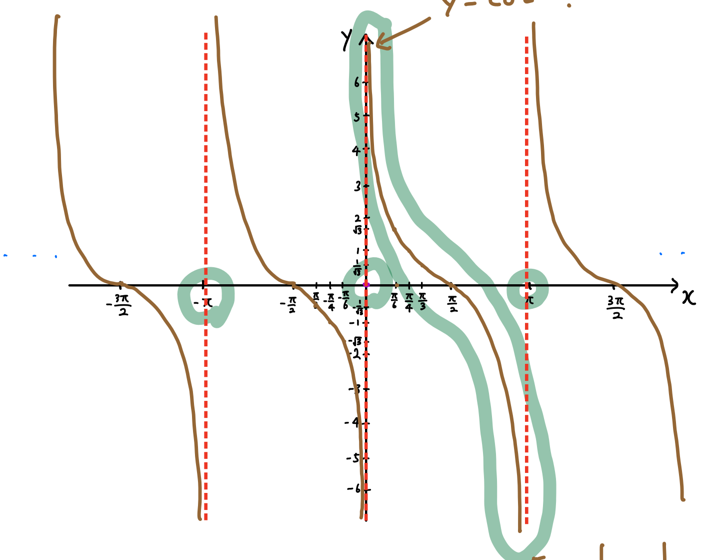
$\tan\frac{\pi}{2} = \frac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}} = \frac{1}{0} =$ undefined.

Using the information about $y = \tan x$, let us try to draw the graph of $y = \cot x$.

$$\cot x = \frac{1}{\tan x} = (\tan x)^{-1} = \left(\frac{\sin x}{\cos x}\right)^{-1} = \frac{\cos x}{\sin x}.$$

Recall : $\cot x = \dfrac{\cos x}{\sin x}$

$y = \cot x$.



← a branch of $y = \cot x$ on $(0, \pi)$.

$y = \cot x$

Domain : $\mathbb{R} - \{\ldots, -2\pi, -\pi, 0, \pi, 2\pi, \ldots\}$

Period : π

Some values of $\cot x$

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y = \cot x$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$y = \tan x$	undefined	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

$$\cot 0 = \frac{\cos 0}{\sin 0} = \frac{1}{0} = \text{undefined}$$

$$\cot \frac{\pi}{2} = \frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}} = \frac{0}{1} = 0.$$

Theorem on the graph of $y = a\tan(bx+c)$

If $y = a\tan(bx+c)$ for nonzero real numbers a, b and c, then

(1) the period is $\dfrac{\pi}{|b|}$.

(2) successive vertical asymptotes for the graph of one branch may be found by solving the inequality

$$\boxed{-\frac{\pi}{2} < bx+c < \frac{\pi}{2}}$$

* We cannot argue about the 'amplitude'.

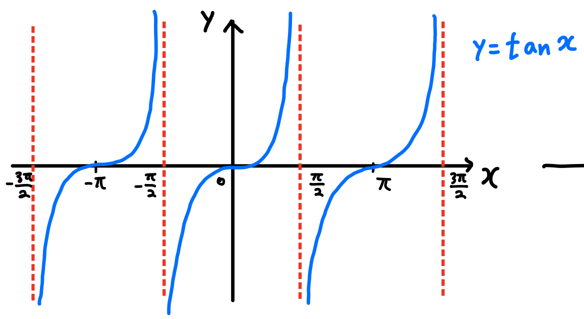
the height from the center line to peak.

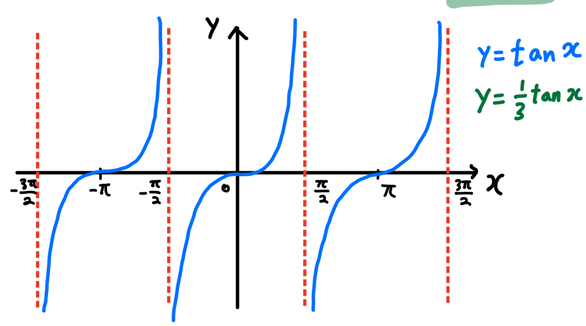
Ex Sketch the graph of $y = \underbrace{\frac{1}{3}}_{a} \tan\left(\underbrace{2}_{b}x - \underbrace{\frac{\pi}{2}}_{c}\right)$

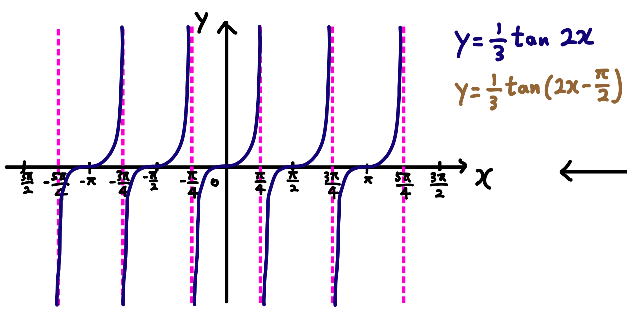
From the above theorem, the period is $\frac{\pi}{|b|} = \frac{\pi}{|2|} = \boxed{\frac{\pi}{2}}$.

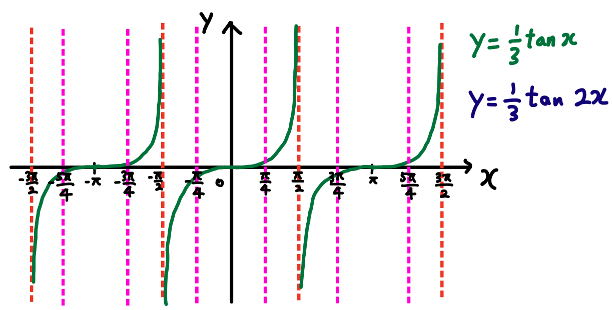
Also, $-\frac{\pi}{2} < 2x - \frac{\pi}{2} < \frac{\pi}{2} \Rightarrow 0 < 2x < \pi$, $0 < x < \frac{\pi}{2}$.

Hence one branch of the tangent function is on $(0, \frac{\pi}{2})$ / We stopped here









* In fact, you can draw the graph directly using the theorem.

$y = \frac{1}{3}\tan(2x - \frac{\pi}{2})$: period $\Rightarrow$

$-\frac{\pi}{2} < 2x - \frac{\pi}{2} < \frac{\pi}{2} \Rightarrow$