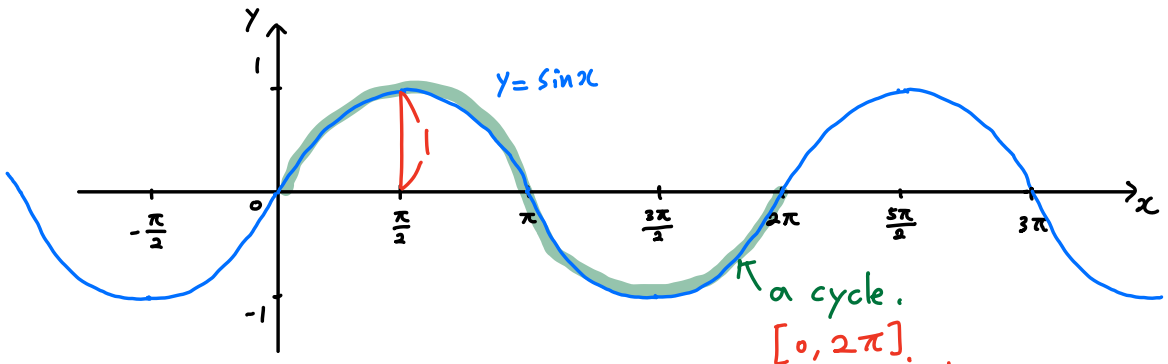
Exam 3 : 11/11 (next Thursday)
HW 9 : due this Friday at 1pm.

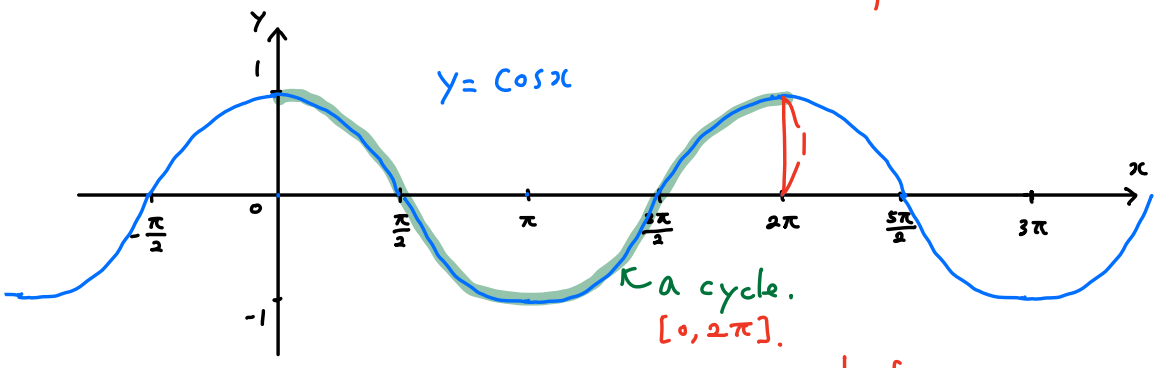
Section 6.5 Trigonometric Graphs (Sine and Cosine)

$y = \sin x$ $y = \cos x$

Recall : The graph of $y = \sin x$ and $y = \cos x$.



a cycle.
$[0, 2\pi]$.
2π : a period of $y = \sin x$.



a cycle.
$[0, 2\pi]$.
2π : a period of $y = \cos x$.

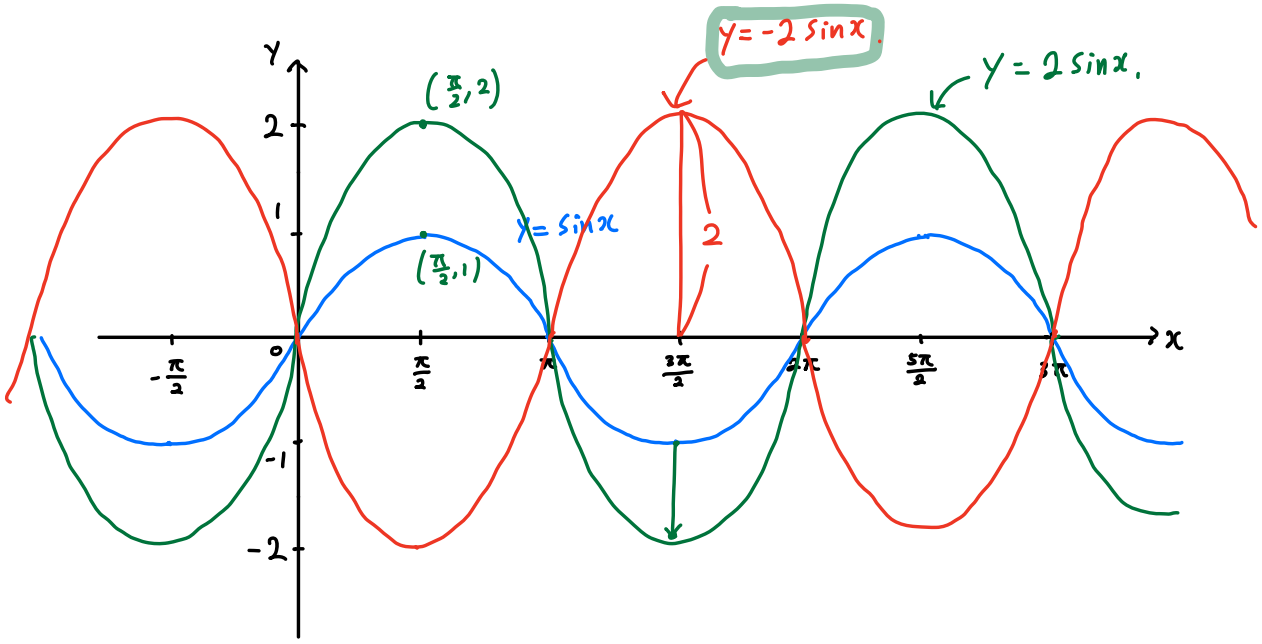
$y = \sin x \longrightarrow y = a \sin x \longrightarrow y = a \sin bx$

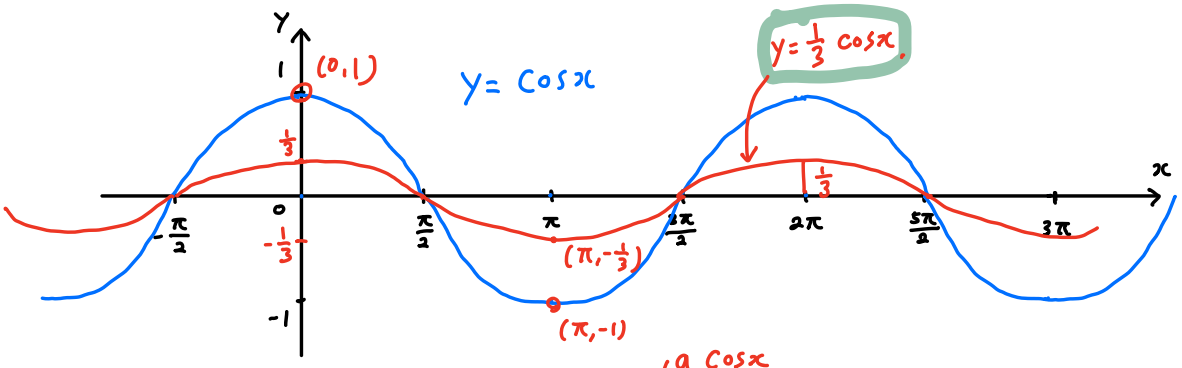
Our goal is to draw the graph of the functions of the forms $y = a \sin(bx+c)$ and $y = a \cos(bx+c)$.

Recall: $y = f(x) \longrightarrow y = af(x)$ ⇒ stretch vertically by a factor a if $a > 1$

Compress vertically by a factor $\frac{1}{a}$ if $0 < a < 1$

Ex Sketch the graph of $y = -2\sin x$ and $y = \frac{1}{3}\cos x$



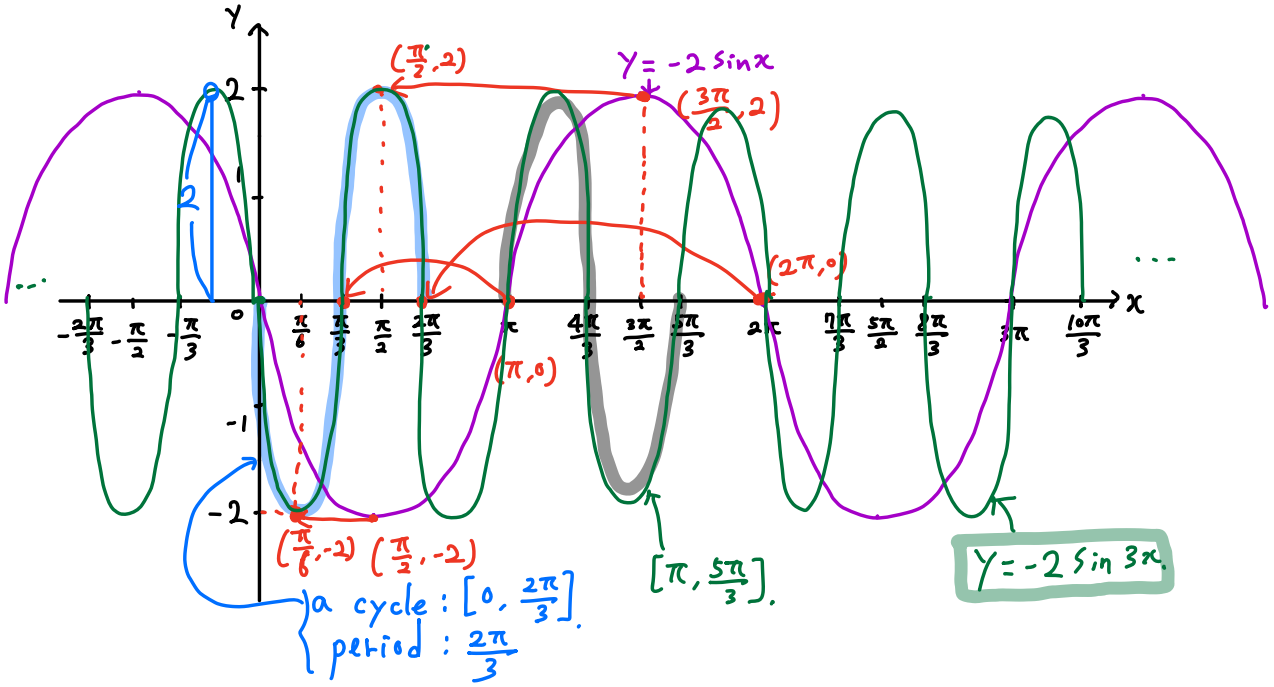


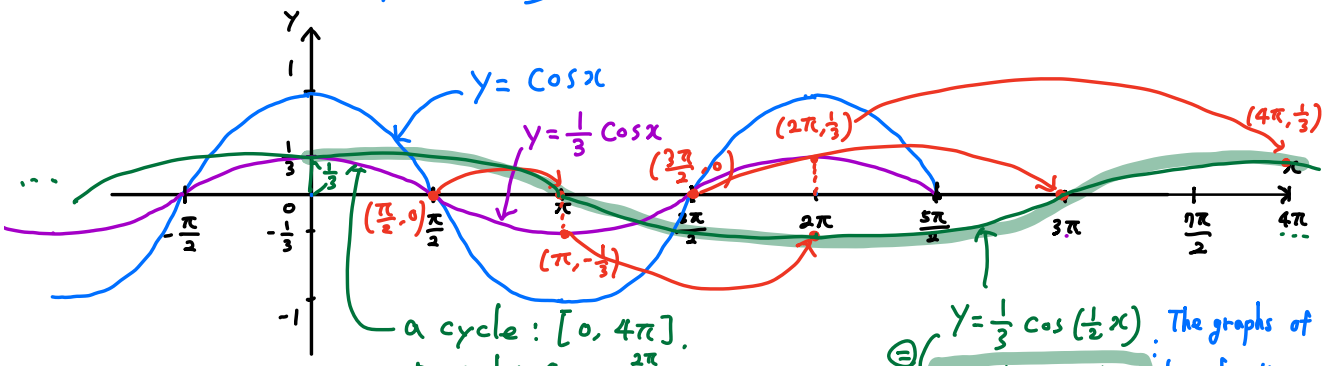
* 'Amplitude of the function $f(x) = \begin{matrix} a\cos x \\ a\sin x \end{matrix}$' is the height from the center line to the peak ⇒ It is $|a|$.

Recall: $y = f(x) \longrightarrow y = f(bx) \Rightarrow$ compress horizontally by a factor b if $b > 1$; stretch horizontally by a factor $\frac{1}{b}$ if $0 < b < 1$

Ex Sketch the graph of $y = -2\sin 3x$ and $y = \frac{1}{3}\cos\left(-\frac{1}{2}x\right)$

$y = \sin x \longrightarrow y = -2\sin x \longrightarrow y = -2\sin 3x$





$y = \cos x \longrightarrow y = \frac{1}{3}\cos x \longrightarrow y = \frac{1}{3}\cos\left(\frac{1}{2}x\right) \longrightarrow y = \frac{1}{3}\cos\left(-\frac{1}{2}x\right)$

$\frac{1}{\frac{1}{2}} = 2.$

$y = \frac{1}{3}\cos\left(\frac{1}{2}x\right) = y = \frac{1}{3}\cos\left(-\frac{1}{2}x\right)$ The graphs of two functions are the same because the graph of $y = \frac{1}{3}\cos\left(\frac{1}{2}x\right)$ is symmetric through y-axis!

As we have observed, the periods of the graphs were no longer 2π. In fact, the following is true:

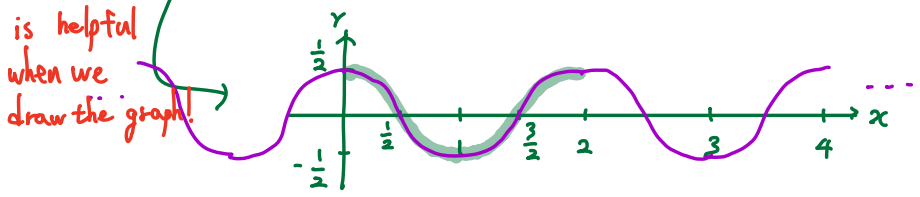
Theorem on Amplitudes and Periods.
If $y = a\sin bx$ or $y = a\cos bx$ for nonzero real numbers a and b, then the graph has amplitude $|a|$, and period $\frac{2\pi}{|b|}$.

Ex The amplitudes and periods of $y = -4\sin\frac{1}{3}x$ and $y = \frac{1}{2}\cos \pi x$ are as follows:

$y = -4\sin\frac{1}{3}x \Rightarrow$ amplitude $= 4$, period $= \frac{2\pi}{\frac{1}{3}} = 6\pi$.

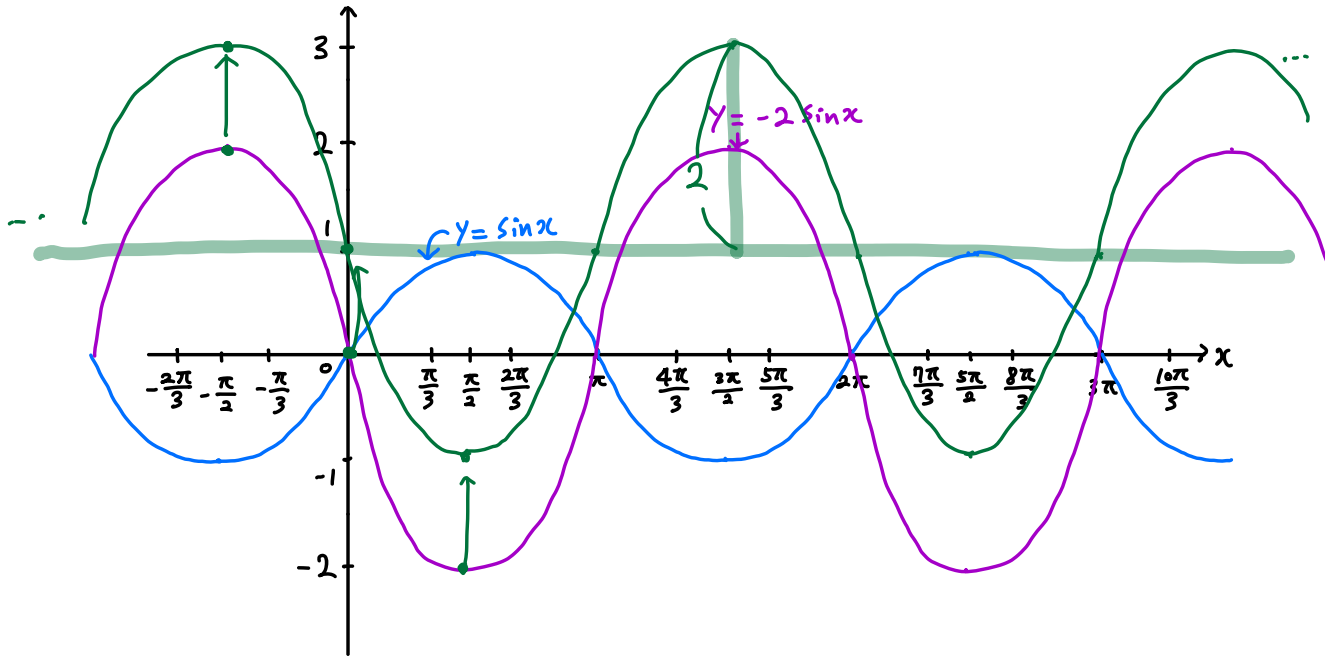
$y = \frac{1}{2}\cos \pi x \Rightarrow$ amplitude $= \frac{1}{2}$, period $= \frac{2\pi}{\pi} = 2$.

These information is helpful when we draw the graph.



Recall : $y = f(x) \longrightarrow y = f(x) + c \Rightarrow$ Vertical shift by c.

Ex Sketch the graph of $y = -2\sin x + 1$



* The amplitude of $y = -2\sin x + 1$ is $|-2| = 2$.
(center line is also shifted !)

$y = a\sin(bx+c)$

$y = a\cos(bx+c)$

Recall: $y = f(x) \to f(x-c) \Rightarrow$ horizontal shift by c.

$y = -2\sin 3x$ shift the graph $\frac{\pi}{2}$ unit to the right!

Ex Sketch $y = -2\sin\left(3x - \frac{3\pi}{2}\right) = -2\sin 3\left(x - \frac{\pi}{2}\right)$