Section 6.3. Continued

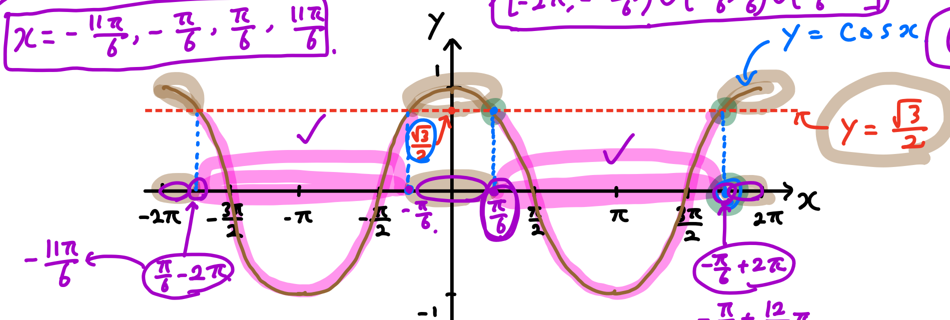
Ex Find all values of x in the interval $[-2\pi, 2\pi]$ such that

(a) $\cos x = \frac{\sqrt{3}}{2}$ (b) $\cos x > \frac{\sqrt{3}}{2}$ (c) $\cos x < \frac{\sqrt{3}}{2}$

(a) $x = -\frac{11\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}$.

(b) $\left[-2\pi, -\frac{11\pi}{6}\right) \cup \left(-\frac{\pi}{6}, \frac{\pi}{6}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right]$

(c) $\left(-\frac{11\pi}{6}, -\frac{\pi}{6}\right) \cup \left(\frac{\pi}{6}, \frac{11\pi}{6}\right)$



$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \rightarrow \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

- Cosine function is $\boxed{\text{even}}$: $\cos(-t) = \cos t$.
- Cosine function has period 2π. : $\cos(t + 2\pi) = \cos t$.

$\cos\left(-\frac{\pi}{6} + 2\pi\right) = \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

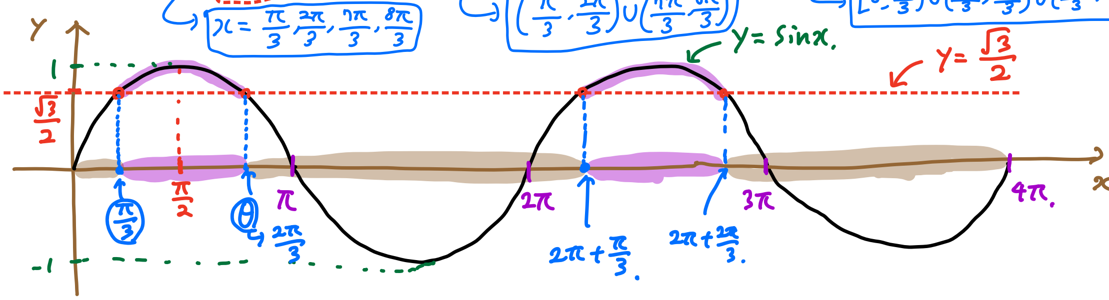
Ex Find all values of x in the interval $[0, 4\pi]$ such that

(a) $\sin x = \frac{\sqrt{3}}{2}$ (b) $\sin x > \frac{\sqrt{3}}{2}$ (c) $\sin x < \frac{\sqrt{3}}{2}$

(a) $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$

(b) $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right) \cup \left(\frac{7\pi}{3}, \frac{8\pi}{3}\right)$

(c) $\left[0, \frac{\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \frac{7\pi}{3}\right) \cup \left(\frac{8\pi}{3}, 4\pi\right]$



We know $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Claim : $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

Why? : $\theta_R = \frac{\pi}{3}$, $\theta = \frac{2\pi}{3}$

$\sin\frac{2\pi}{3}$ is positive, so $\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$!

Also, since Sine is 2π-periodic,

$2\pi + \frac{\pi}{3}$ and $2\pi + \frac{2\pi}{3}$ are also solutions.

$= \frac{7\pi}{3}$ $= \frac{8\pi}{3}$

Section 6.4 Values of the Trigonometric Functions.

(and in M211, M212)

In this course, you should be able to find the values of the trigonometric functions for varius angles.

We can achieve this goal by using

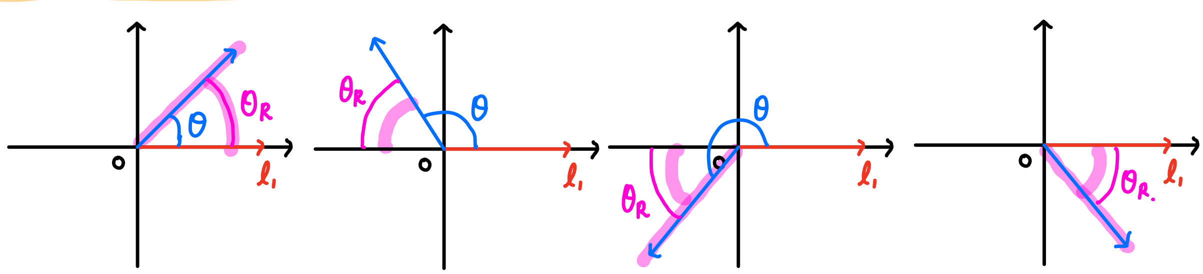
① the preodicity of the trigonometric functions

and ② 'reference angles.'

angle whose terminal side is not on either x-axis or y-axis.

Reference Angle

Let θ be a nonquadrantal angle in standard position.
The reference angle for θ is the acute angle θ_R that the terminal side of θ makes with the x-axis.



Theorem on Reference Angles

If θ is a nonquadrantal angle in standard position, then to find the value of a trigonometric function at θ, find its value for the reference angle θ_R and prefix the appropriate sign.

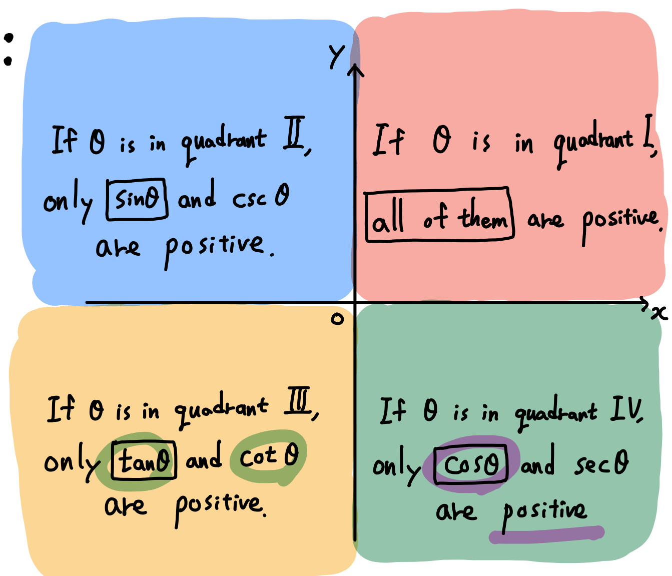
$\cos\theta$? $\longrightarrow$ 1) find the reference angle θ_R

2) Find $\cos\theta_R$

3) $\cos\theta$ is either $\cos\theta_R$ or $-\cos\theta_R$

Q. How to prefix the sign?

A. Use it:



Ex Find the exact values :

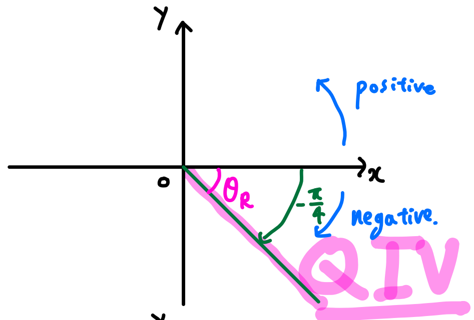
1) $\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \left(\text{or } \frac{\sqrt{2}}{2}\right)$

2) $\sin 240^\circ = -\frac{\sqrt{3}}{2}$

3) $\sec\left(\frac{29\pi}{6}\right) = -\frac{2}{\sqrt{3}}$

4) $\cot\left(\frac{5\pi}{3}\right) = -\frac{1}{\sqrt{3}}$

Proof : 1)

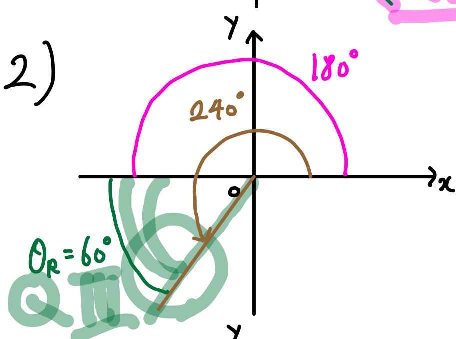


$\theta = -\frac{\pi}{4}$, $\theta_R = \frac{\pi}{4}$

$\hookrightarrow \cos\theta_R = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

$\cos\theta = \cos\left(-\frac{\pi}{4}\right) = \boxed{\frac{1}{\sqrt{2}}}$ or $\cancel{-\frac{1}{\sqrt{2}}}$

2)

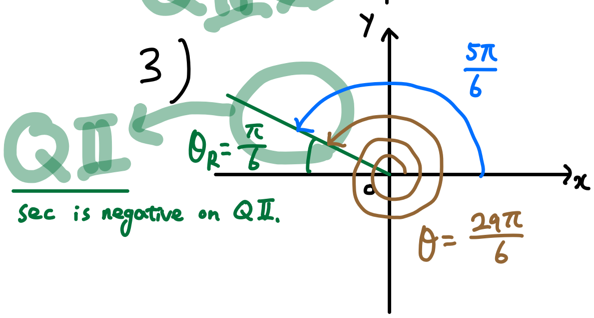


$\theta = 240^\circ$, $\theta_R = 60^\circ$

$\hookrightarrow \sin\theta_R = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\sin 240^\circ = \cancel{\frac{\sqrt{3}}{2}}$ or $\boxed{-\frac{\sqrt{3}}{2}}$.

3)



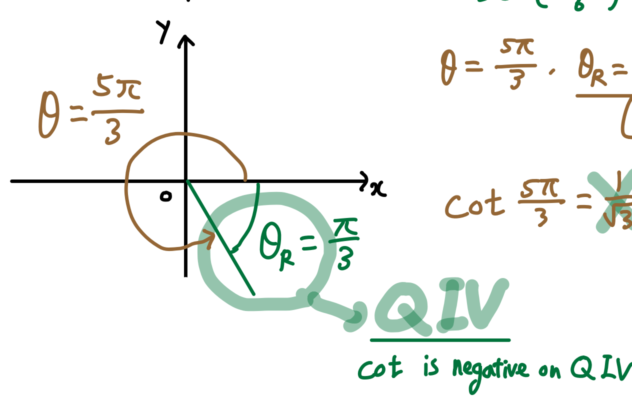
$\frac{29\pi}{6} = \frac{12\pi}{6} + \frac{12\pi}{6} + \frac{5\pi}{6} = 2\pi + 2\pi + \frac{5\pi}{6}$

$\Rightarrow \sec\left(\frac{29\pi}{6}\right) = \sec\left(\frac{5\pi}{6}\right)$

$\theta = \frac{5\pi}{6}$, $\theta_R = \frac{\pi}{6}$ $\hookrightarrow \sec\theta_R = \sec\frac{\pi}{6} = \frac{1}{\cos\frac{\pi}{6}} = \frac{2}{\sqrt{3}}$

$\sec\left(\frac{29\pi}{6}\right) = \cancel{\frac{2}{\sqrt{3}}}$ or $\boxed{-\frac{2}{\sqrt{3}}}$

4)



$\theta = \frac{5\pi}{3}$, $\theta_R = \frac{\pi}{3}$

$\hookrightarrow \cot\theta_R = \cot\frac{\pi}{3} = \frac{1}{\tan\frac{\pi}{3}} = \frac{1}{\sqrt{3}}$

$\cot\frac{5\pi}{3} = \cancel{\frac{1}{\sqrt{3}}}$ or $\boxed{-\frac{1}{\sqrt{3}}}$