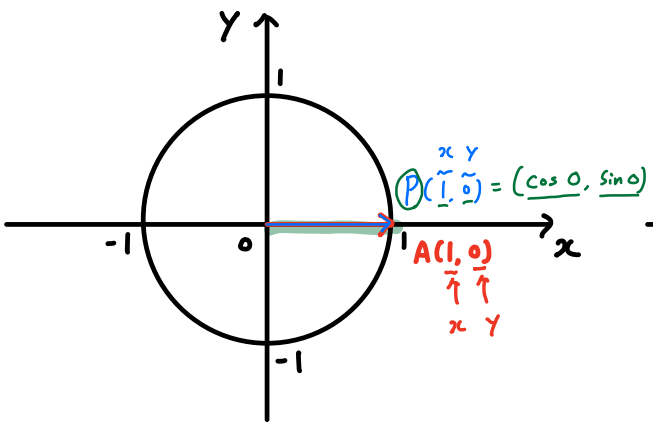
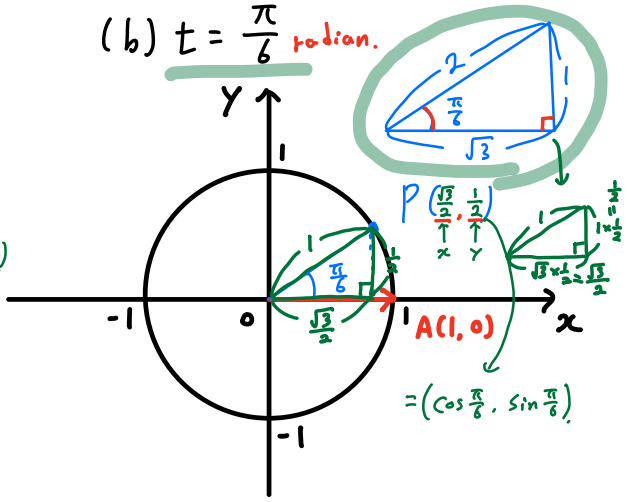
Do the HW 8: due tomorrow at 1 p.m.

**Ex** Find the values of the trigonometric functions at $t$.

(a) $t = 0$ radian.




(b) $t = \frac{\pi}{6}$ radian.




Recall

$\sin t = y$ , $\cos t = x$ , $\tan t = \frac{y}{x}$ (if $x \neq 0$)

$\csc t = \frac{1}{y}$ (if $y \neq 0$) , $\sec t = \frac{1}{x}$ (if $x \neq 0$) , $\cot t = \frac{x}{y}$ (if $y \neq 0$)

(a) $x = 1$, $y = 0$

$\sin 0 = 0$, $\cos 0 = 1$, $\tan 0 = \frac{0}{1} = 0$

$\csc 0 = \frac{1}{0}$ undefined, $\sec 0 = 1$, $\cot 0 = \frac{1}{0}$ undefined.

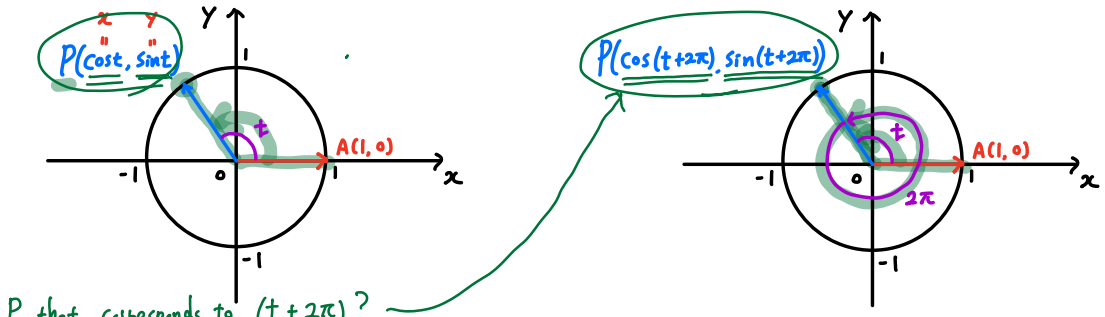
(b) $x = \frac{\sqrt{3}}{2}$, $y = \frac{1}{2}$

$\sin \frac{\pi}{6} = \frac{1}{2}$, $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\tan \frac{\pi}{6} = \frac{\frac{1}{2} \times 2}{\frac{\sqrt{3}}{2} \times 2} = \frac{1}{\sqrt{3}}$

$\csc \frac{\pi}{6} = \frac{1}{\sin \frac{\pi}{6}} = 2$

$\sec \frac{\pi}{6} = \frac{1}{\cos \frac{\pi}{6}} = \frac{2}{\sqrt{3}}$

$\cot \frac{\pi}{6} = \frac{1}{\tan \frac{\pi}{6}} = \sqrt{3}$



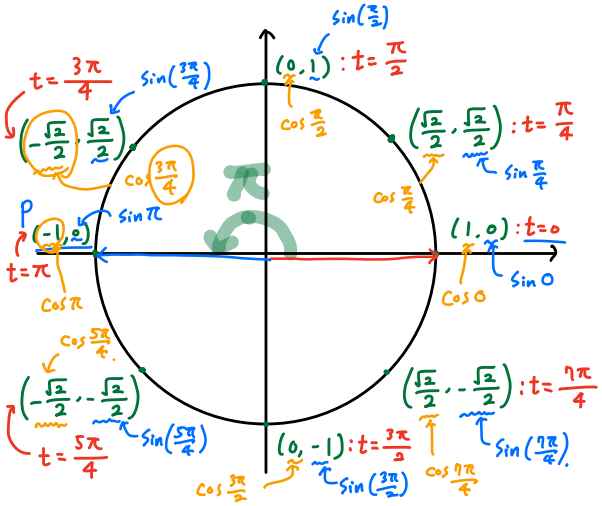

P that corresponds to $(t+2\pi)$?

* $P(\cos t, \sin t)$ and $P(\cos(t+2\pi), \sin(t+2\pi))$ are the same point!

Hence, we have

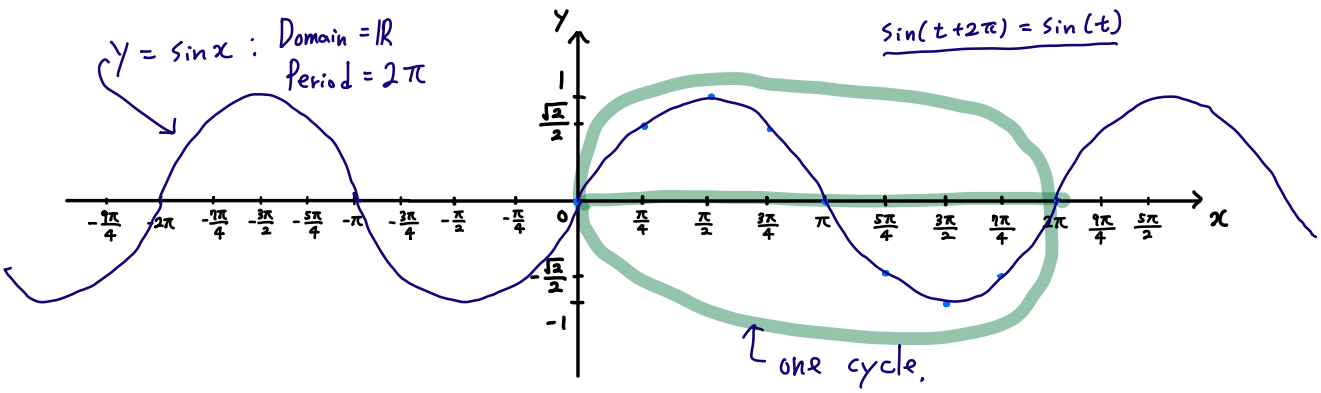
$$\sin(t+2\pi) = \sin t \quad \text{and} \quad \cos(t+2\pi) = \cos t.$$

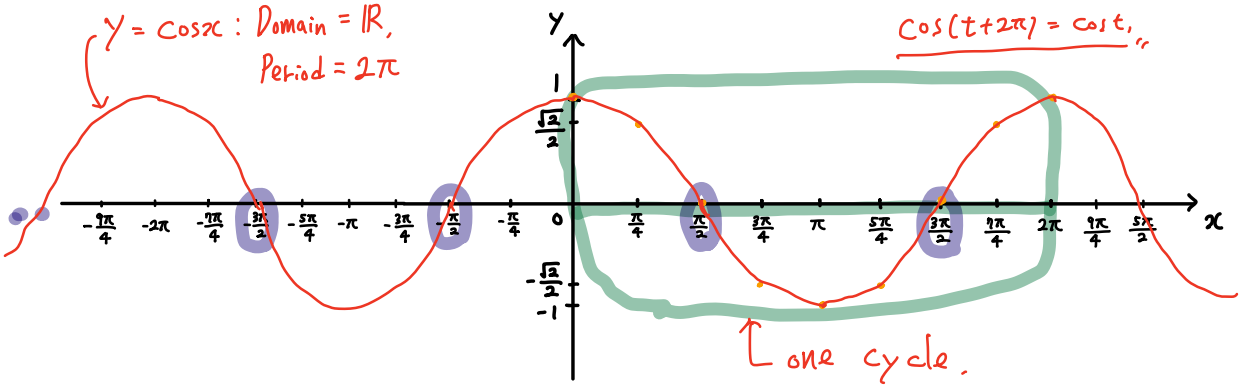
Now, we want to draw the graph of $y = \sin x$ and $y = \cos x$.




| $x$ | $0$ | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ | $\frac{5\pi}{4}$ | $\frac{3\pi}{2}$ | $\frac{7\pi}{4}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $y = \sin x$ | $0$ | $\frac{\sqrt{2}}{2}$ | $1$ | $\frac{\sqrt{2}}{2}$ | $0$ | $-\frac{\sqrt{2}}{2}$ | $-1$ | $-\frac{\sqrt{2}}{2}$ | $0$ |

| $x$ | $0$ | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ | $\frac{5\pi}{4}$ | $\frac{3\pi}{2}$ | $\frac{7\pi}{4}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $y = \cos x$ | $1$ | $\frac{\sqrt{2}}{2}$ | $0$ | $-\frac{\sqrt{2}}{2}$ | $-1$ | $-\frac{\sqrt{2}}{2}$ | $0$ | $\frac{\sqrt{2}}{2}$ | $1$ |




From the graph of $y = \sin x$ and $y = \cos x$, we can see that

$y = \sin x$ is odd, and $y = \cos x$ is even.

($f(-x) = -f(x)$ ↷ odd; $f(-x) = f(x)$ ↷ even.)

$\Rightarrow$ $\sin(-x) = -\sin x$, $\cos(-x) = \cos x$.

$\tan(-t) = \dfrac{\sin(-t)}{\cos(-t)} = \dfrac{-\sin t}{\cos t} = -\tan t$

Using the fundamental identities, we can show the following:

($f(-t) = f(t)$)

$\sin(-t) = -\sin t$ (odd), $\cos(-t) = \cos t$ (even), $\tan(-t) = -\tan t$ (odd)

$\csc(-t) = -\csc t$ (odd), $\sec(-t) = \sec t$ (even), $\cot(-t) = -\cot t$ (odd)

Thus, we can conclude the following:

(1) The cosine and secant functions are even.

(2) The sine, tangent, cotangent, and cosecant functions are odd.

Using the graph of $y = \sin x$, $y = \cos x$, we can draw the graph of $y = \tan x = \frac{\sin x}{\cos x}$.

$\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$.

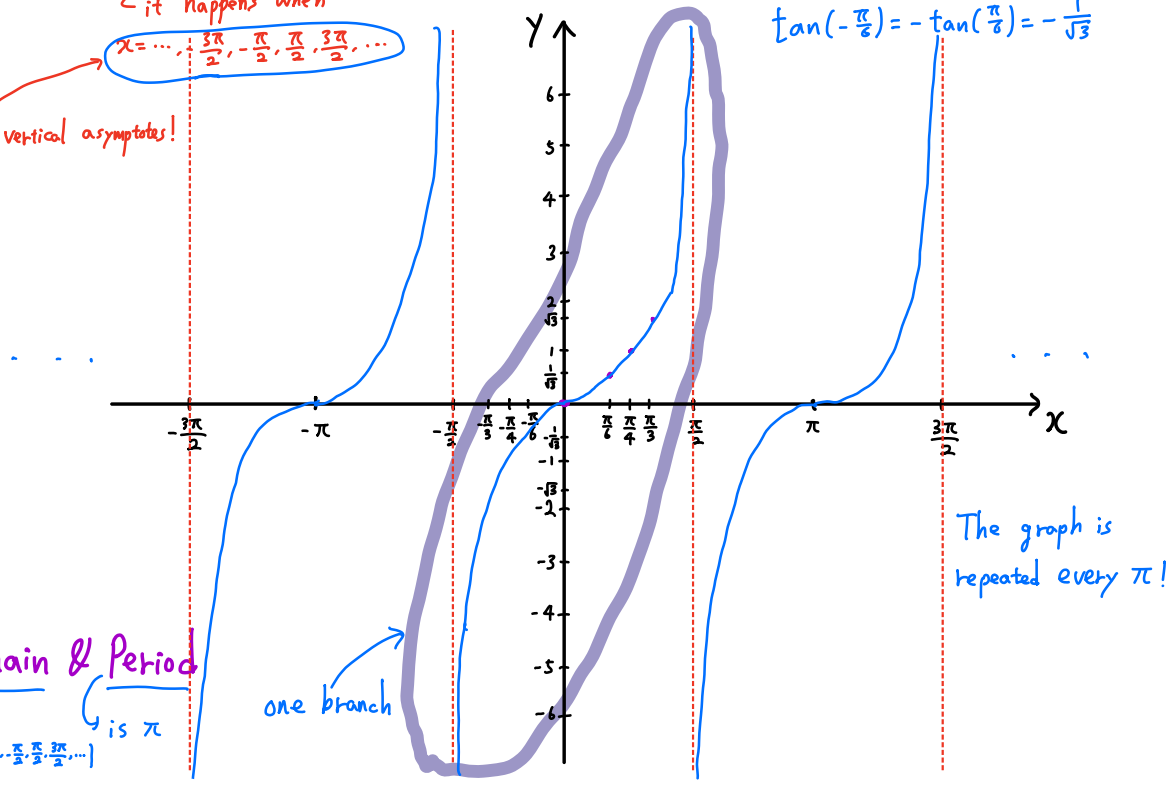
$\tan x$ is undefined when $\cos x = 0$.

it happens when $x = \cdots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \cdots$

we have vertical asymptotes!

| $x$ | $-\frac{\pi}{3}$ | $-\frac{\pi}{4}$ | $-\frac{\pi}{6}$ | $0$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ |
|---|---|---|---|---|---|---|---|
| $y = \tan x$ | $-\sqrt{3}$ | $-1$ | $-\frac{1}{\sqrt{3}}$ | $0$ | $\frac{1}{\sqrt{3}}$ | $1$ | $\sqrt{3}$ |

$\tan(-\frac{\pi}{6}) = -\tan(\frac{\pi}{6}) = -\frac{1}{\sqrt{3}}$




The graph is repeated every $\pi$!

* Domain & Period

is $\mathbb{R} - \{\cdots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \cdots\}$

is $\pi$

one branch