(Section 6.2 Continue)

Sign of the Trigonometric function

If θ is in quadrant II, only $\boxed{\sin\theta}$ and $\csc\theta$ are positive.

If θ is in quadrant I, $\boxed{\text{all of them}}$ are positive.

If θ is in quadrant III, only $\boxed{\tan\theta}$ and $\cot\theta$ are positive.

If θ is in quadrant IV, only $\boxed{\cos\theta}$ and $\sec\theta$ are positive

Why ? when $P(x,y)$

$$\sin\theta = \frac{y}{r} = \frac{y\text{-coordinate of } P}{\sqrt{x^2+y^2}} \leftarrow \text{positive}$$

$\sin\theta$ is positive if the y-coordinate of P is positive.

$$\cos\theta = \frac{x}{r} = \frac{x\text{-coordinate of } P}{\sqrt{x^2+y^2}} \leftarrow \text{positive}$$

$\cos\theta$ is positive if the x-coordinate of P is positive

	Q I	Q II	Q III	Q IV
$\sin\theta$	+	+	−	−
$\cos\theta$	+	−	−	+
$\tan\theta$	+	−	+	−

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

Ex If $\cos\theta = \dfrac{5}{13}$ and $\tan\theta < 0$, find the values of the other five trigonometric functions using the fundamental identities.

θ is on Q IV. Why? $\cos\theta = \frac{5}{13} > 0 \Rightarrow \theta$ is either on Q I or Q IV.

$\tan\theta < 0$: θ should be on Q IV

Recall : $\sin^2\theta + \cos^2\theta = 1 \rightarrow \sin^2\theta + \left(\frac{5}{13}\right)^2 = 1$, $\sin^2\theta = 1 - \frac{25}{169} = \frac{144}{169} = \left(\frac{12}{13}\right)^2$, $\sin\theta = \cancel{\frac{12}{13}}$ or $-\frac{12}{13}$

$\sin\theta = -\frac{12}{13}$ $\quad \csc\theta = -\frac{13}{12}$

$\cos\theta = \frac{5}{13}$ $\quad \sec\theta = \frac{13}{5}$

$\tan\theta = -\frac{12}{5}$ $\quad \cot\theta = -\frac{5}{12}$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{-\frac{12}{13}}{\frac{5}{13}} = -\dfrac{12}{5}$

Ex Rewrite the expression in nonradical form without using absolute values for the indicated values of θ.

$\sqrt{1+\tan^2\theta}$; $\pi < \theta < \dfrac{3\pi}{2}$ rad

$\times\frac{180}{\pi}$ $\quad$ $\times\frac{180}{\pi}$

$180^\circ < \theta < 270^\circ \Rightarrow \theta$ is on Q III.

$\sqrt{1+\tan^2\theta} = \sqrt{\sec^2\theta} = |\sec\theta| = \boxed{-\sec\theta}$

used $1+\tan^2\theta = \sec^2\theta$

used $\sqrt{x^2} = |x|$

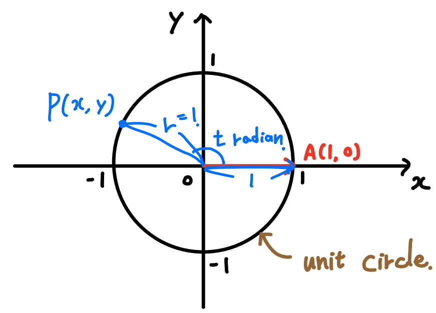
used $|x| = -x$ if $x<0$, and $\sec\theta = \frac{1}{\cos\theta}$ is negative on Q III.

Section 6.3 Trigonometric Functions of Real Numbers.

* Unless you see the degree symbol (ex) $60°$), all the angles θ are understood as θ radian!

Ex $\theta = \frac{\pi}{2} \Rightarrow \theta$ is $\frac{\pi}{2}$ radian.

For any real number t we define the values of the trigonometric functions as follows: (t radian.)



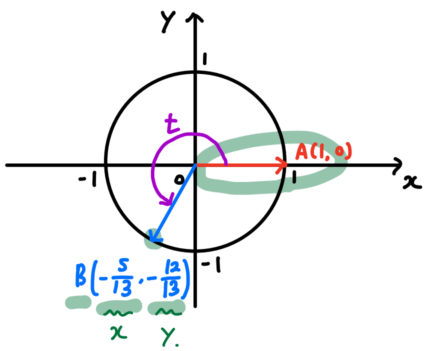
Rotate the radius $\overline{OA}$ t radian with respect to the origin O. Let $P(x,y)$ be the intersection of the terminal side and the unit circle.

Since $P(x,y)$ is on the unit circle, $r =$ 1. Hence,

$\sin t = \frac{y}{\cancel{r}\,1} = \boxed{y}$, $\cos t = \frac{x}{\cancel{r}\,1} = \boxed{x}$, $\tan t = \boxed{\frac{y}{x}}$ (if $x \neq 0$)

$\csc t = \frac{\cancel{r}\,1}{y}$ (if $y \neq 0$) $= \boxed{\frac{1}{y}}$, $\sec t = \frac{\cancel{r}\,1}{x}$ (if $x \neq 0$) $= \boxed{\frac{1}{x}}$, $\cot t = \boxed{\frac{x}{y}}$ (if $y \neq 0$)

Ex A point $P(x, y)$ on the unit circle U corresponding to a real number t is shown below, for $\pi < t < \frac{3\pi}{2}$. Find the values of the trigonometric functions at t.



$\sin t = y = \boxed{-\frac{12}{13}}$ $\qquad$ $\csc t = \frac{1}{y} = \boxed{-\frac{13}{12}}$

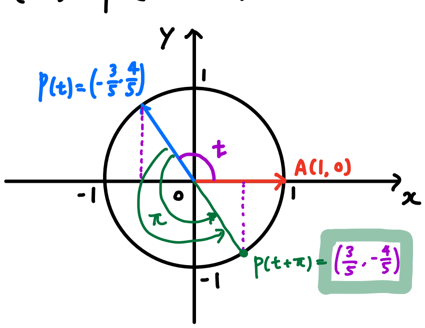
$\cos t = x = \boxed{-\frac{5}{13}}$ $\qquad$ $\sec t = \frac{1}{x} = \boxed{-\frac{13}{5}}$

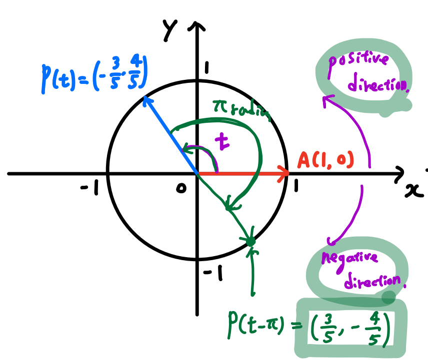
$\tan t = \frac{y}{x}$ $\qquad$ $\cot x = \frac{x}{y} = \boxed{\frac{5}{12}}$

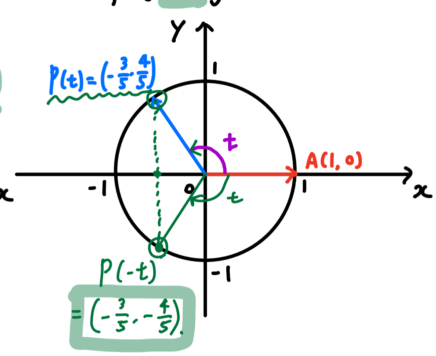
$= \dfrac{-\frac{12}{13}}{-\frac{5}{13}} = \boxed{\frac{12}{5}}$

Ex Let $P(t)$ denote the point on the unit circle U that corresponds to t (radian) for $0 \leq t < 2\pi$. If $P(t) = \left(-\frac{3}{5}, \frac{4}{5}\right)$, find

(a) $P(t+\pi)$ $\qquad$ (b) $P(t-\pi)$ $\qquad$ (c) $P(-t)$

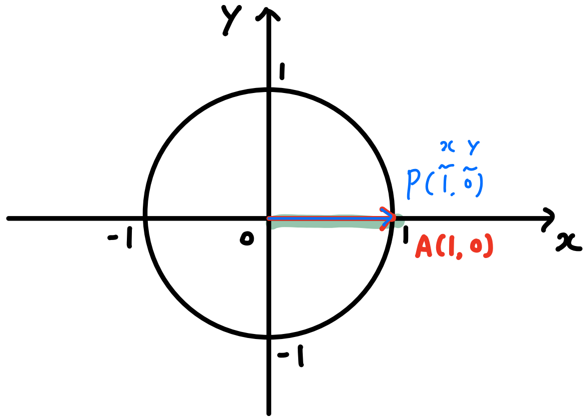




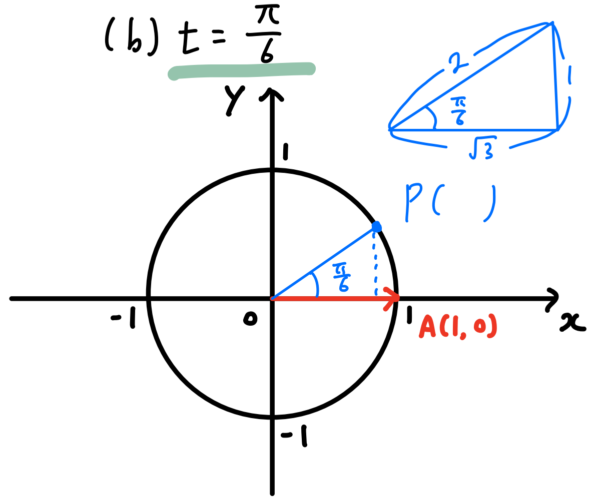


Ex Find the values of the trigonometric functions at t.

(a) $t = 0$



(b) $t = \frac{\pi}{6}$



Recall

$\sin t = y$, $\cos t = x$, $\tan t = \frac{y}{x}$ (if $x \neq 0$)

$\csc t = \frac{1}{y}$ (if $y \neq 0$), $\sec t = \frac{1}{x}$ (if $x \neq 0$), $\cot t = \frac{x}{y}$ (if $y \neq 0$)

(a) $x = 1$, $y = 0$

$\sin 0 = \boxed{0}$, $\cos 0 = \boxed{1}$, $\tan 0 = \frac{0}{1} = \boxed{0}$

$\csc 0 = \cancel{\frac{1}{0}}$ $\boxed{\text{undefined}}$, $\sec 0 = \boxed{1}$, $\cot 0 = \cancel{\frac{1}{0}}$ $\boxed{\text{undefined}}$,