HW 8 is due next Friday at 1pm

# Section 5.6. Exponential and Logarithmic equations.

Ex) $\log_7 11 = \dfrac{\log_3 11}{\log_3 7}$

## Change of base Formula

If $u > 0$ and if $a$ and $b$ are positive real numbers different from 1, then $\log_b u = \dfrac{\log_a u}{\log_a b}$.

Proof) $b^{\log_b u} = u$ : Recall $\log_a u^c = c \cdot \log_a u$

$\log_a b^{\boxed{\log_b u}} = \log_a u$

$$\frac{\log_b u \cdot \cancel{\log_a b}}{\cancel{\log_a b}} = \frac{\log_a u}{\log_a b} \Rightarrow \boxed{\log_b u = \frac{\log_a u}{\log_a b}}$$

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Recall : $y = \log_a z$ if $z = a^y$

Ex Solve $5^x = 17$

proof 1) $17 = 5^x \longrightarrow \boxed{x = \log_5 17}$

$\log_5 17 = \dfrac{\log_e 17}{\log_e 5} = \dfrac{\ln 17}{\ln 5}$

$= \dfrac{\log_{11} 17}{\log_{11} 5}$

proof 2) $5^x = 17$

$\ln 5^x = \ln 17$

$x \cdot \ln 5 = \ln 17$

$\boxed{x = \dfrac{\ln 17}{\ln 5}}$

They are all the same answers!

Ex Solve $7^{3x-1} = 4^{5x+2}$

Take ln!

$\ln 7^{3x-1} = \ln 4^{5x+2}$

$(3x-1) \cdot \ln 7 = (5x+2) \cdot \ln 4$

$3 \cdot \ln 7 \cdot x - \cancel{\ln 7} = 5 \cdot \cancel{\ln 4 \cdot x} + 2 \cdot \ln 4$

$- 5 \cdot \ln 4 \cdot x \qquad - 5 \cdot \cancel{\ln 4 \cdot x}$

$+ \cancel{\ln 7} \qquad + \ln 7$

$3 \cdot \ln 7 \, x - 5 \cdot \ln 4 \cdot x = 2 \cdot \ln 4 + \ln 7$

$\dfrac{x \cancel{(3\ln 7 - 5\ln 4)}}{\cancel{(3\ln 7 - 5\ln 4)}} = \dfrac{2\ln 4 + \ln 7}{(3\ln 7 - 5\ln 4)}$, $\boxed{x = \dfrac{2\ln 4 + \ln 7}{3\ln 7 - 5\ln 4}}$

Recall : $2^{3x-1} = 4^{5x+2}$

$2^{3x-1} = (2^2)^{5x+2}$

$2^{3x-1} = 2^{10x+4}$

$3x - 1 = 10x + 4$

$7x = -5$, $\boxed{x = -\frac{5}{7}}$

We cannot use this approach to the problem $7^{3x-1} = 4^{5x+2}$!

Ex Solve $\dfrac{3^x - 3^{-x}}{5} = 2$

$\downarrow \times 5$

$3^x - 3^{-x} = 10$

$3^x - \dfrac{1}{3^x} = 10$

$\downarrow \times 3^x$

$(3^x)^2 - 1 = 10 \cdot 3^x$

$(3^x)^2 - 10 \cdot 3^x - 1 = 0.$

Let $A = 3^x$. Then,

$A^2 - 10A - 1 = 0.$ Quadratic Formula!

$$A = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1}$$

$$= \frac{10 \pm \sqrt{100+4}}{2}$$

$$= \frac{10 \pm \sqrt{104}}{2}$$

$$= \frac{10 \pm \sqrt{4} \cdot \sqrt{26}}{2}$$

$$= \frac{10 \pm 2\sqrt{26}}{2}$$

$A = \dfrac{10 + 2\sqrt{26}}{2}$ or $\dfrac{10 - 2\sqrt{26}}{2}$.

$A = 5 + \sqrt{26}$ or $5 - \sqrt{26}$

$3^x = \underbrace{5 + \sqrt{26}}_{pos}$ or $\cancel{\underbrace{5 - \sqrt{26}}_{neg.}}$

$3^x = (5 + \sqrt{26})$

$\boxed{x = \log_3 (5 + \sqrt{26})}$

Recall $5^x = 17$ if $x = \log_5 17$

Ex Solve $\log \sqrt[4]{x} = \sqrt{\log x}$

$\log x^{\frac{1}{4}} = (\log x)^{\frac{1}{2}} \neq \log x^{\frac{1}{2}}$

$\frac{1}{4} \log x = (\log x)^{\frac{1}{2}}$

Let $B = (\log x)^{\frac{1}{2}}$

$\frac{1}{4} \cdot B^2 = B \xrightarrow{\times 4} B^2 = 4B$

$\rightarrow B^2 - 4B = 0$

$\rightarrow B(B-4) = 0.$

$\downarrow$ Z.F.T.

$B = 0$ or $B = 4.$

$(\log x)^{\frac{1}{2}} = 0$ or $4.$

$(\log x)^{\frac{1}{2}} = 0, \quad (\log x)^{\frac{1}{2}} = 4.$

$\log_{10} x = 0 \quad \log_{10} x = 16.$

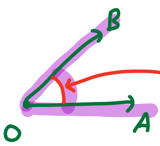
$x = 10^0, \quad x = 10^{16}$

$\boxed{x = 1, \ x = 10^{16}}$

# Chapter 6. The Trigonometric Functions.

## Section 6.1. Angles.

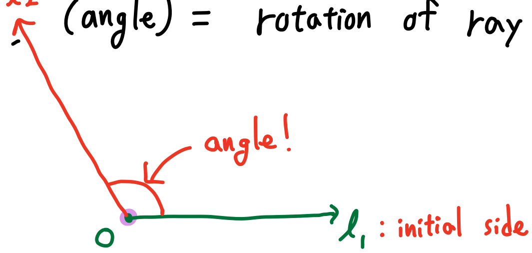
Angle is determined by two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ that share a vertex O. :



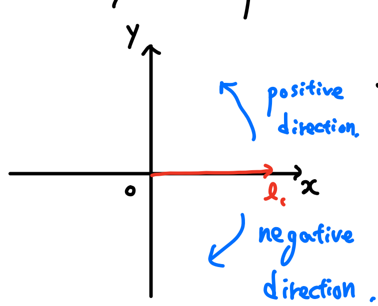

angle $\boxed{\angle AOB}$

: terminal side.

In trigonometry $l_2$ : (angle) = 'rotation of ray'




Usually we put O on the origin of the coordinate plane, $l_1$ on the positive $x$-axis.

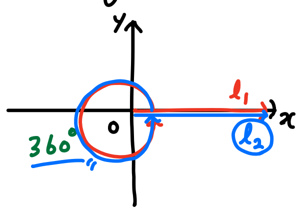



Counterclockwise : positive angle

clockwise : negative angle.

If the ray $l_1$ turn around once in the counterclockwise direction, the angle determined by the two rays is 360 degree

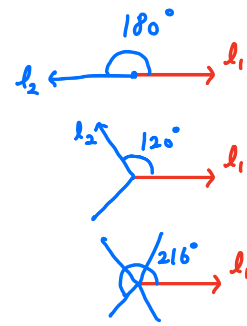
($360^\circ$).




The half of $360^\circ$ is $360^\circ \times \frac{1}{2} = 180^\circ$;

A third of $360^\circ$ is $360^\circ \times \frac{1}{3} = 120^\circ$;

Three-fifth of $360^\circ$ is $360^\circ \times \frac{3}{5} = 216^\circ$,

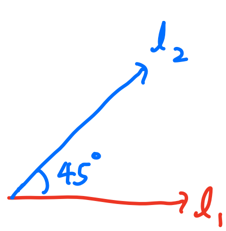


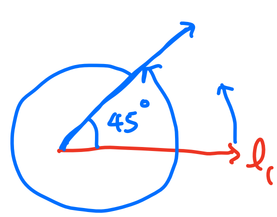

. . . .

If two angles differ by $360^\circ$ or its multiple, they looks the same! (because their terminal side coincide).

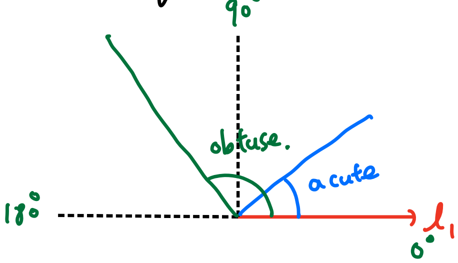
Such angles are called "coterminal angles".

Ex $\theta = 45^\circ$ $\qquad$ $\theta = 45^\circ + 360^\circ = 405^\circ$



An angle $\theta$ is called $\begin{cases} \text{"acute" if } 0 < \theta < 90^\circ \\ \text{"obtuse" if } 90^\circ < \theta < 180^\circ \end{cases}$




Two angles $\alpha$ and $\beta$ are $\begin{cases} \text{"complementary angles" if } \alpha + \beta = 90^\circ. \\ \text{"supplementary angles" if } \alpha + \beta = 180^\circ. \end{cases}$

(1 inch $= \frac{1}{12}$ feet.)

* Smaller measurements : 1 minute $= \boxed{\frac{1}{60} \text{ degree}}$ (1 degree = 60 minutes)

$$1' = \frac{1}{60}^\circ \qquad 1^\circ = 60'$$

1 second $= \boxed{\frac{1}{60} \text{ minute}}$ (1 minute = 60 seconds)

$$1'' = \frac{1}{60}' \qquad 1' = 60''$$

Ex $\theta = 34^\circ 42' 15''$ is

$$\hookrightarrow 34^\circ + 42' + 15'' = 34^\circ + \frac{42}{60}^\circ + \frac{15}{60}' = \boxed{34^\circ + \frac{42}{60}^\circ + \frac{15}{60 \cdot 60}^\circ}$$

The angle complementary to $\theta$ is

$$90^\circ - \theta = 90^\circ - 34^\circ 42' 15''$$

$$\begin{aligned} 90^\circ &= 89^\circ + 1^\circ \\ &= 89^\circ + 60' \\ &= 89^\circ + 59' + 1' \\ &= 89^\circ + 59' + 60'' = \boxed{89^\circ 59' 60''} \end{aligned}$$

$$\begin{array}{r} 89^\circ\ 59'\ 60'' \\ -\ 34^\circ\ 42'\ 15'' \\ \hline \boxed{55^\circ\ 17'\ 45''} \end{array}$$