Section 5.2. Continued

HW7 : due tonight : 11:59 pm.
HW8 : will be posted this Friday.

Ex Sketch the graph of ① $f(x) = 2^x$

$f(0) = 2^0 = 1 \to (0,1)$ $\quad f(-1) = 2^{-1} = \frac{1}{2} \to (-1, \frac{1}{2})$

$f(1) = 2^1 = 2 \to (1,2)$

② $f(x) = \left(\frac{3}{2}\right)^x$

$f(-1) = \left(\frac{3}{2}\right)^{-1} = \frac{2}{3}$ $(-1, \frac{2}{3})$

$f(0) = \left(\frac{3}{2}\right)^0 = 1 \to (0,1)$

$f(1) = \left(\frac{3}{2}\right)^1 = \frac{3}{2} \to (1, \frac{3}{2})$

③ $f(x) = \left(\frac{1}{2}\right)^x$

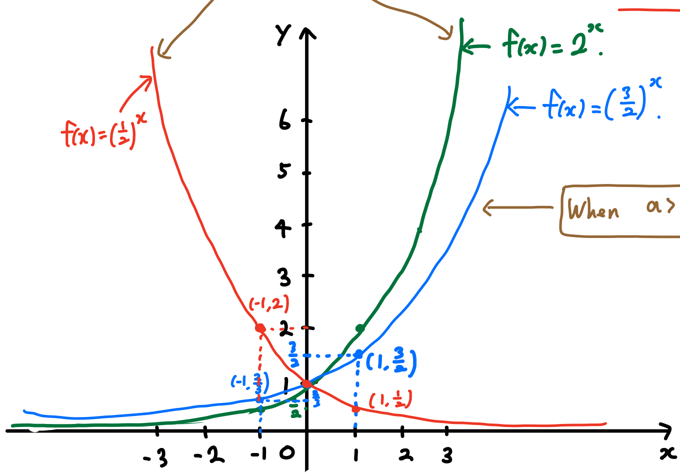
$f(-1) = \left(\frac{1}{2}\right)^{-1} = 2 \to (-1, 2)$

$f(0) = \left(\frac{1}{2}\right)^0 = 1 \to (0,1)$

$f(1) = \left(\frac{1}{2}\right)^1 = \frac{1}{2} \to (1, \frac{1}{2})$

$= \left(\frac{1}{2}\right)^x = 2^{-x}$: obtained from ① by replacing $x \to -x$.

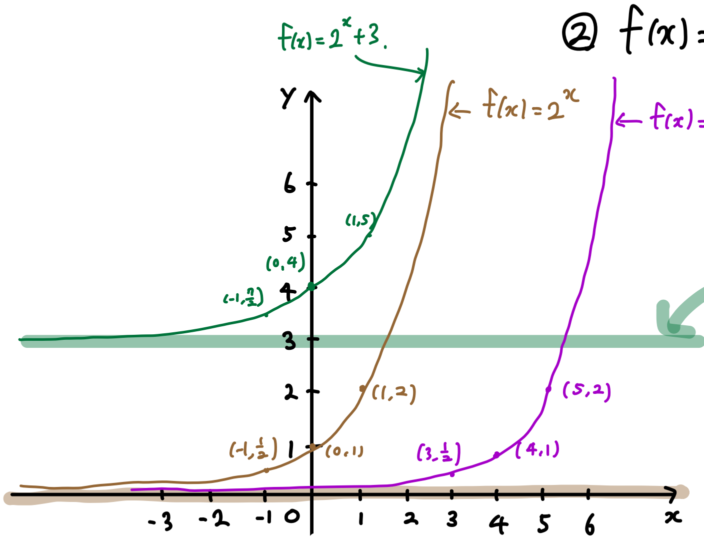
⇒ Graphs of ① and ③ are symmetric with respect to y-axis!



Ex Sketch the graph of ① $f(x) = 2^{x-4}$

$\boxed{f(x) = 2^x}$ x by $x-4$

② $f(x) = 2^x + 3$



$y=3$: The horizontal asymptotes of the function $f(x) = 2^x + 3$.

Ex Find an exponential function of the form $f(x)=ba^{-x}+c$ that has horizontal asymptote $y=-4$, y-intercept 12 and x-intercept 2.

↳ (0, 12) is on the graph ↳ (2, 0) is on the graph.

① $f(x)=b\cdot a^{-x}+c$ is obtained from $f(x)=b\cdot a^{-x}$ by adding c.

↳ $\boxed{y=c}$ should be the horizontal asymptote!

Hence, $c=-4$: $f(x)=ba^{-x}-4$.

base of an exponential function should be a positive number not equal to 1.

② Since (0, 12) is on the graph,

$y=ba^{-x}-4 \xrightarrow{x=0,\,y=12} 12=b\cdot a^{0}-4$

$\Rightarrow 12=b\cdot 1-4$

$\Rightarrow b=16$: $f(x)=16a^{-x}-4$

③ Since (2, 0) is on the graph,

$y=16a^{-x}-4 \xrightarrow{x=2,\,y=0} 0=16\cdot a^{-2}-4$

$\Rightarrow 16a^{-2}=4$

$\Rightarrow a^{-2}=\frac{1}{4}$

$\frac{1}{a^2}=\frac{1}{4} \Rightarrow a^2=4$

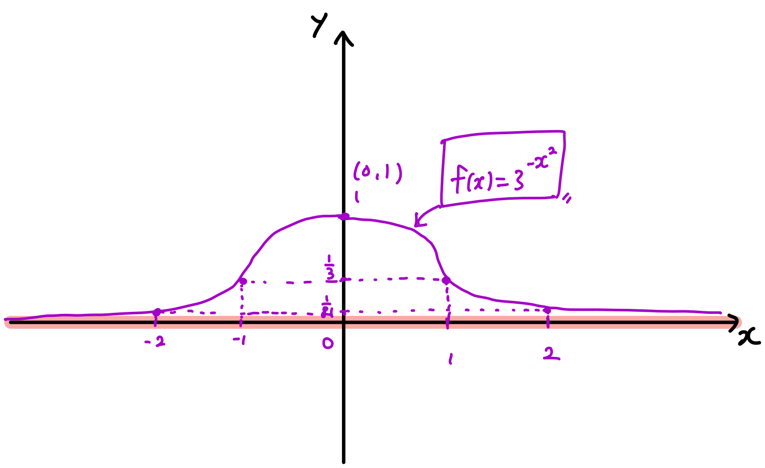
$\Rightarrow a=2$ or $\cancel{a=-2}$.

Hence, $\boxed{f(x)=16\cdot 2^{-x}-4}$

Ex Sketh the graph of $f(x)=3^{-x^2}$.

$f(x)=3^{-x^2}$, $f(-x)=3^{-(-x)^2}=3^{-x^2}=f(x)$: f is even.

↳ The graph is symmetric w.r.t. y-axis



$f(0)=3^{-0^2}=3^{0}=1 \rightarrow (0,1)$

$f(1)=3^{-1^2}=3^{-1}=\frac{1}{3} \rightarrow (1,\frac{1}{3})$

$f(2)=3^{-2^2}=3^{-4}=\frac{1}{3^4}=\frac{1}{81} \rightarrow (2,\frac{1}{81})$

$\underbrace{(-1)^2}_{1} \neq \underbrace{-1^2}_{-1}$

Application : Compound Interest

* Recall : Simple Interest.

Suppose $\$P$ is the principal (= amount of money that was initially invested)

and r is the interest rate.

If the interest is simple interest, we earn $\$Pr$ each year.

Hence, after t year, the total amount of interest (I) is

$I = Prt$ and the total amount of money becomes $P + Prt$.

Ex If $P = \$1,000$ and the interest rate (Simple interest) is 20%

$0.20 = r$

Now — 1 year later — 2 years later — 3 years later

Now: $\$1,000$

1 year later: $(\text{interest}) = P \cdot r = 1,000 \cdot 0.20 = \$200$

2 years later: $(\text{interest}) = P \cdot r = 1,000 \cdot 0.20 = \$200$

3 years later: $(\text{interest}) = P \cdot r = 1,000 \cdot 0.20 = \$200$

$$I = \underbrace{\$1,000}_{P} \cdot \underbrace{0.20}_{r} \cdot \underbrace{3}_{t} = \$600 \Rightarrow (\text{Total amount of money})$$

$= \$1,000 + \$1,000 \cdot 0.20 \cdot 3$

$= \$1,600$

Compound Interest : We update the principal as soon as we get interest!

Suppose P is the principal (= amount of money that was initially invested) and r is the interest rate.

Also, suppose the interest is compounded $\boxed{\text{once}}$ a year.

Then after t years, the total amount of money is $\boxed{P\cdot(1+r)^t}$

$$\boxed{\$1000\cdot(1+0.2)^3 = \$1,728}$$

Ex If $P = \$1,000$ and the interest rate (compound interest) is 20%. $r = 0.20$.

Now	1 year later	2 years later	3 years later
$P = \$1,000$.	$I = \$1,000\cdot 0.20$	$I = \$1,200\cdot 0.20$	$I = 1,440\cdot 0.20$
	$= \$200$	$= \$240$	$= \$288$
	new P : $1,000+200$	new P : $1,200+240$	new P : $1,440+288$
	$= 1,200$	$= 1,440$	$= \boxed{\$1,728}$
	$= 1,000\cdot(1+0.20)^1$	$= 1,200\cdot(1+0.20)^1$	$= 1,440\cdot(1+0.20)^1$
		$= 1,000\cdot(1+0.20)^1\cdot(1+0.20)^1$	$= 1,000\cdot(1+0.20)^2\cdot(1+0.20)^1$
		$= 1,000\cdot(1+0.20)^2$	$= 1,000\cdot(1+0.20)^3$