Section 5.1 continued

* HW7 is due this Sunday at 11:59pm.
* HW8 will be posted next Friday

Ex Let $f(x) = \dfrac{5x-3}{3x+2}$. Find $f^{-1}(x)$.

one-to-one function!

Check that
1. x-intercept : $\left(\frac{3}{5}, 0\right)$
2. Vertical asymptote : $x = -\frac{2}{3}$
3. y-intercept : $\left(0, -\frac{3}{2}\right)$
4. horizontal asymptote : $y = \frac{5}{3}$

Domain of f: $\mathbb{R} - \{-\frac{2}{3}\}$

Range of f: $\mathbb{R} - \{\frac{5}{3}\}$

Domain of $f^{-1} = \mathbb{R} - \{\frac{5}{3}\}$

Range of $f^{-1} = \mathbb{R} - \{-\frac{2}{3}\}$

$f(x) = \dfrac{5x-3}{3x+2}$

$y = \dfrac{5x-3}{3x+2}$: Solve for x.

$\big\downarrow \times(3x+2)$

$y\cdot(3x+2) = 5x-3$

$3xy + 2y = 5x - 3$

$\quad -5x \quad -5x$

$3xy + 2y - 5x = -3$

$\quad -2y \quad -2y$

$3xy - 5x = -2y - 3$

$x(3y-5) = -2y-3$

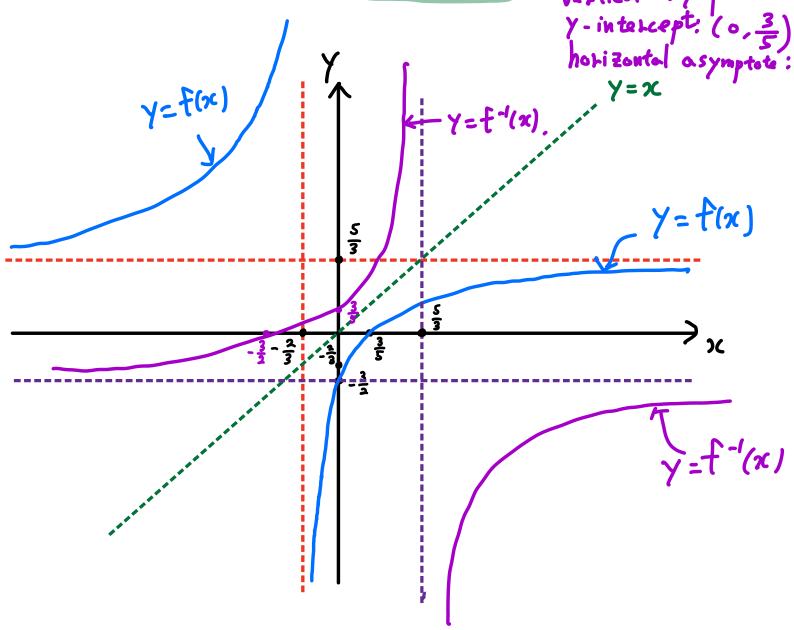
$\big\downarrow \div(3y-5)$

$x = \dfrac{-2y-3}{3y-5}$: interchange x and y.

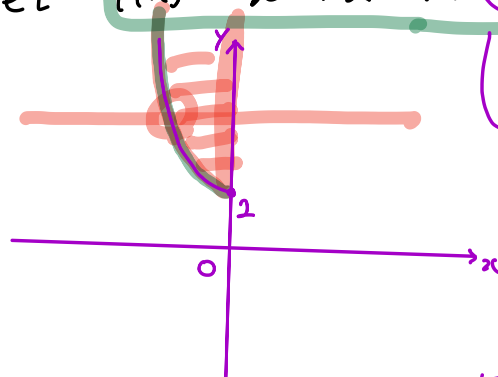
$\Rightarrow y = \dfrac{-2x-3}{3x-5}$: replace y by $f^{-1}(x)$

$-2x-3=0$

$\Rightarrow \boxed{f^{-1}(x) = \dfrac{-2x-3}{3x-5}}$:
x-intercept $\left(-\frac{3}{2}, 0\right)$
vertical asymptote : $x = \frac{5}{3}$
y-intercept : $\left(0, \frac{3}{5}\right)$
horizontal asymptote : $y = -\frac{2}{3}$



Ex Let $f(x) = x^2 + 2$ for $x \le 0$. Find $f^{-1}(x)$.



Domain of f: $(-\infty, 0]$.

Range of f: $[2, \infty)$

Domain of $f^{-1} = [2, \infty)$

Range of $f^{-1} = (-\infty, 0]$.

$f(x) = x^2 + 2$, $x \le 0$.

$y = x^2 + 2$: Solve for x

$-2 \qquad -2$

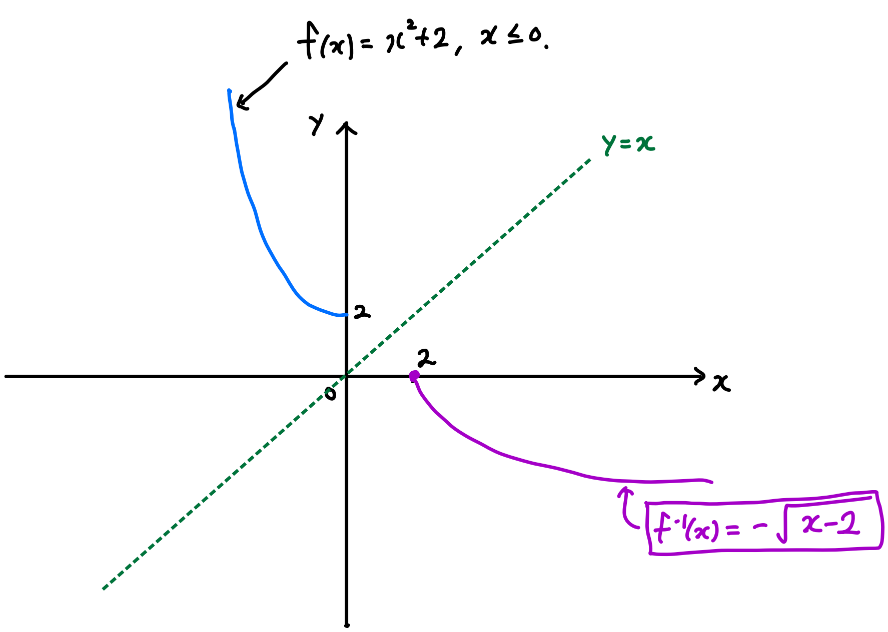
$y - 2 = x^2$

$x^2 = y - 2 \Rightarrow \cancel{x = +\sqrt{y-2}}$ or $x = -\sqrt{y-2}$: interchange x and y.

because $x \le 0$

$y = -\sqrt{x-2}$: replace y by $f^{-1}(x)$

$\boxed{f^{-1}(x) = -\sqrt{x-2}}$ //



Section 5.2 Exponential Functions.

- $2^4 = 2\cdot2\cdot2\cdot2 = 16$
- $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ $\qquad : a^{-n} = \frac{1}{a^n}$
- $2^{\frac{7}{3}} = \sqrt[3]{2^7} = \sqrt[3]{2^6\cdot 2} = \sqrt[3]{2^6}\cdot\sqrt[3]{2} = \sqrt[3]{(2^2)^3}\cdot\sqrt[3]{2} = 2^2\cdot\sqrt[3]{2}$
  $= 4\cdot\sqrt[3]{2}$ $\qquad : a^{\frac{m}{n}} = \sqrt[n]{a^m}$
- $2^{\sqrt{2}} = \qquad 2^1,\ 2^{1.4} = 2^{\frac{14}{10}} = \sqrt[10]{2^{14}},\ 2^{1.41},\ 2^{1.414},\ 2^{1.4142},\ \cdots \rightarrow 2^{\sqrt{2}}$

  $* \sqrt{2} = 1.4142\cdots.$

  the numbers get closer and closer to some real number, and we define $2^{\sqrt{2}}$ to be the real number.

For any $a>0$ and $a\neq 1$, we consider a function

$\boxed{f(x) = a^x}$ $\qquad f(2) = a^2$

"Exponential function f with base a."

(Recall from Section 1.2)

* Law of exponents still holds when m and n are any real numbers.

① $a^m a^n = a^{m+n}$ $\qquad 2^{\pi}\cdot 2^{\sqrt{3}} = 2^{\pi+\sqrt{3}}$

④ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

⑦ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

② $(a^m)^n = a^{mn}$

⑤ $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$ $\qquad .\ a^{-n} = \frac{1}{a^n}$

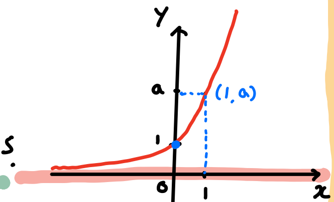
③ $(ab)^n = a^n b^n$

⑥ $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$

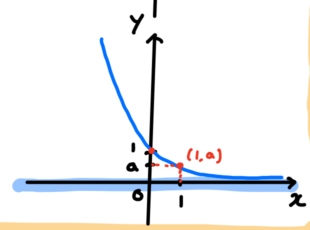
$f(x)=a^x$: $f(0)=a^0=1 \rightarrow (0,1)$
$f(1)=a^1=a \rightarrow (1,a)$

* Properties of exponential function

① If $a>1$, the function $f(x)=a^x$ is increasing and x-axis is horizontal asymptotes.



② If $0<a<1$, the function $f(x)=a^x$ is decreasing and x-axis is horizontal asymptotes.



Since the exponential function $f(x)=a^x$ is either increasing or decreasing, it is one-to-one.

Thus, for real numbers x_1 and x_2, the following equivalent conditions are satisfied : (1) If $x_1 \neq x_2$, then $a^{x_1} \neq a^{x_2}$
$f(x_1) \neq f(x_2)$.

(2) If $a^{x_1} = a^{x_2}$, then $x_1 = x_2$.
$f(x_1) = f(x_2)$

We can solve exponential equations using these properties.

Ex ① Solve $3^{4x+7} = 3^{2x+1}$ → Exponents should be the same,

$4x+7 = 2x+1 \Rightarrow 2x=-6$, $\boxed{x=-3}$

② Solve $2^{4x+7} = 8^{x+4}$

$8^{x+4} = (2^3)^{x+4} = 2^{3\cdot(x+4)}$

used

$(a^m)^n = a^{mn}$

$2^{4x+7} = 2^{3\cdot(x+4)}$: Exponents should be the same.

$\Downarrow$

$4x+7 = 3(x+4)$

$4x+7 = 3x+12$

$\boxed{x=5}$