Exam 2 : Section 3.2 – Section 4.6.
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Section 3.1

Section 4.5. Rational Functions.

A function $f(x)$ is a rational function

if $f(x) = $ (rational expression)

$\Rightarrow f(x) = \frac{g(x)}{h(x)}$ where $g(x)$ and $h(x)$ are polynomials.

Ex) $f(x) = \frac{2x+3}{x^2-x-6}$

Q: Domain of $f(x) = \frac{g(x)}{h(x)}$?

A: All real numbers – (every x that makes $h(x) = 0$)

Ex $f(x) = \frac{3x+5}{x^2-4}$ has domain $\mathbb{R} - \{\pm 2\}$ or $(-\infty, \infty) - \{\pm 2\}$

$x^2-4 \to h(x)$.

When $h(x) = 0$? $x^2 - 4 = 0. \to x^2 - 2^2 = 0$, $(x+2)(x-2) = 0 \xrightarrow{\text{Z.F.T.}} x+2=0$ or $x-2=0$

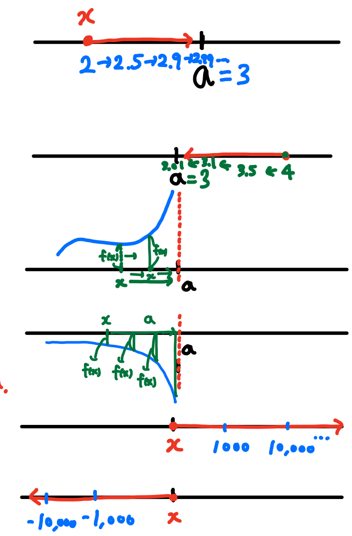
$x+2=0 \Rightarrow x=-2$, $x-2=0 \Rightarrow x=2$.

The goal of this section is to draw the graph of rational functions.

To do that, we should know the behavior of $f(x)$

when
- x is close to a zero of the denominator.
- x is large positive or large negative.

Notation	Terminology
$x \longrightarrow a^-$	x approaches a from the left
$x \longrightarrow a^+$	x approaches a from the right
$f(x) \longrightarrow \infty$ as $x \longrightarrow a$	$f(x)$ increase without bound as x approaches a
$f(x) \longrightarrow -\infty$ as $x \longrightarrow a$	$f(x)$ decrease without bound as x approaches a.
$x \longrightarrow \infty$	x increase without bound
$x \longrightarrow -\infty$	x decrease without bound

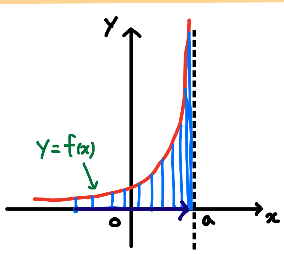


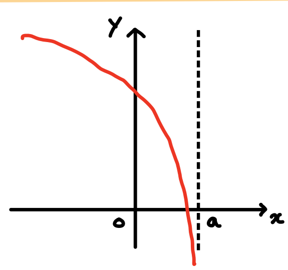
In many case the graph of rational functions has "Vertical Asymptote" or "Horizontal Asymptote"

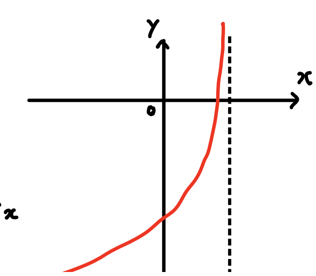
The line $x = a$ is a vertical asymptote for the graph of a function f if

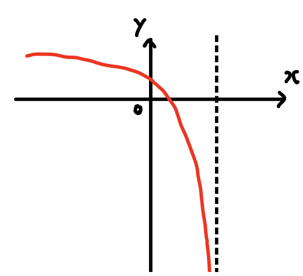
$$f(x) \longrightarrow \infty \quad \text{or} \quad f(x) \longrightarrow -\infty$$

as x approaches a from either the left or the right.



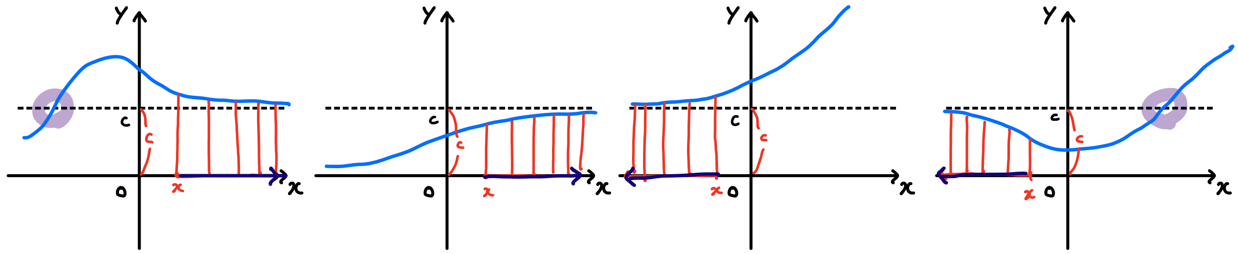






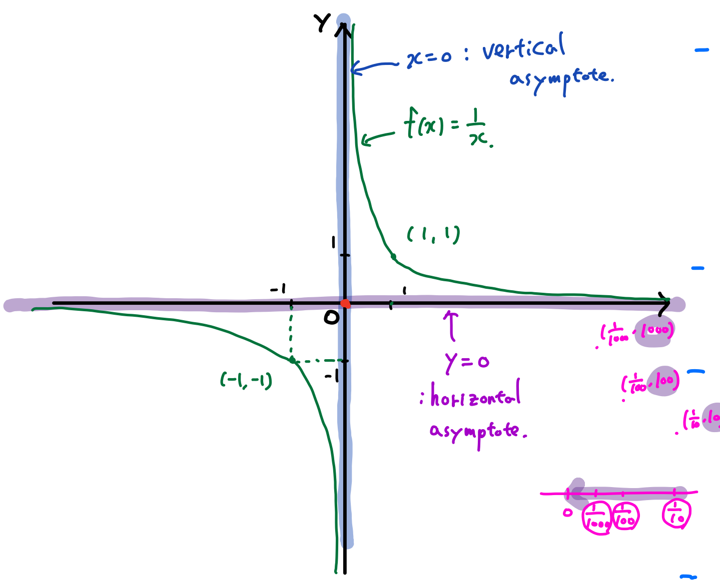
The line $y=c$ is a horizontal asymptote for the graph of a function f if

$$\boxed{f(x) \longrightarrow c} \text{ as } x \longrightarrow \infty \text{ or as } x \longrightarrow -\infty.$$



* Horizontal asymptote may intersect with the graph of the function!

The simplest rational function : $\boxed{f(x)=\frac{1}{x}}$ ← when $x=1$, $f(1)=1$.



- Domain : $\mathbb{R}-\{0\}$, $(-\infty,\infty)-\{0\}$
  $(-\infty,0)\cup(0,\infty)$

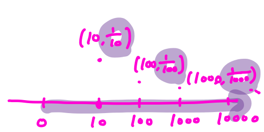
$f(-x)=\frac{1}{-x}=-\frac{1}{x}=-f(x)$, so $f(x)$ is odd function

- On $(0,\infty)$ $f(x)=\frac{1}{x}$ is decreasing.
  $6 \longmapsto \frac{1}{6}$
  $10 \longmapsto \frac{1}{10}$
- As $x \longrightarrow 0^+$, $f(x) \longrightarrow \infty$.

x	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$
$f(x)$	10	100	1000	10,000

$f(x)=\frac{1}{x}$ $\quad f(\frac{1}{10})=\frac{1}{\frac{1}{10}}=\frac{\frac{1}{1}}{\frac{1}{10}}=\frac{10\cdot1}{1\cdot1}=10$ $\quad \left(\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{ad}{bc}\right)$

- As $x \longrightarrow \infty$, $f(x) \longrightarrow 0$



x	10	100	1,000	10,000
$f(x)$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10,000}$

$\frac{1}{x}$

Using Section 3.5, we also know the graph of the function $\boxed{f(x) = \frac{1}{x-a} + b}$:

Ex) $f(x) = \frac{1}{x}$

$\downarrow$ translate the graph 3 units to the right.

$f(x) = \frac{1}{x-3}$.

$\downarrow$ translate the graph 2 units up.

$f(x) = \frac{1}{x-3} + 2$