Section 4.1 Continued.

HW5 : due this Saturday at 11:59pm

Sketching the graph of a polynomial function of degree ≥ 3.

Facts : When x approaches to ∞ or $-\infty$, any polynomial function $f(x)$ approaches to ∞ or $-\infty$.

Ex Sketch the graph of $f(x) = x^3 - 3x^2 - x + 3 = (x-3)(x+1)(x-1)$.

STEP1. Factor! $x^3 - 3x^2 - x + 3$

$= x^2(x-3) - (x-3)$

$= (x-3)(x^2 - 1^2)$

$= (x-3)(x+1)(x-1)$

STEP2 Find x-intercept : Solve $x^3 - 3x^2 - x + 3 = 0$

$\Rightarrow (x-3)(x+1)(x-1) = 0$

$\downarrow$ Z.F.T.

$x-3=0$ or $x+1=0$ or $x-1=0$

$\downarrow$

$x=3, x=-1, x=1$

$\downarrow$

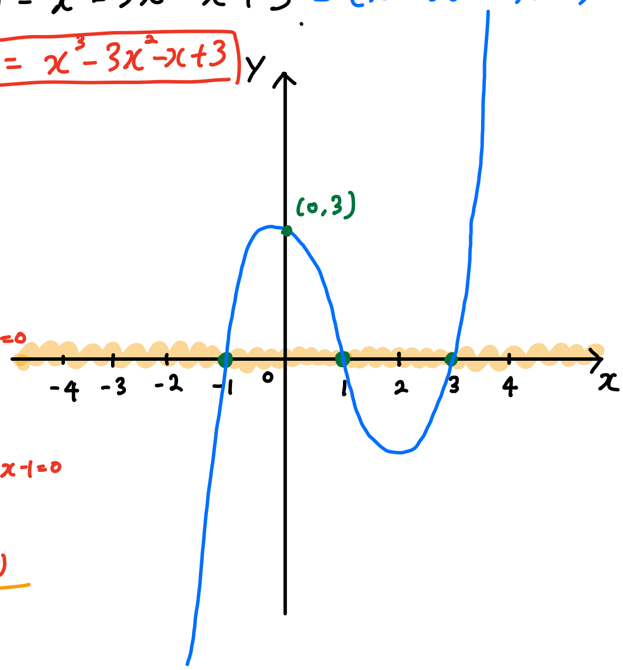
$(3,0), (-1,0), (1,0)$

y-intercept : $(0, 3)$

STEP3 Analyze the sign of $f(x) = (x-3)(x+1)(x-1)$

	$(-\infty,-1)$	-1	$(-1,1)$	1	$(1,3)$	3	$(3,\infty)$
$f(x)=(x-3)(x+1)(x-1)$	⊖		⊕		⊖		⊕
$(x-3)$	−		−		−	0	+
$(x+1)$	−	0	+		+		+
$(x-1)$	−		−	0	+		+

$y = x^3 - 3x^2 - x + 3$



Ex Sketch the graph of $f(x) = x^4 - x^3 - 6x^2$.

STEP1 Factor : $f(x) = x^4 - x^3 - 6x^2$

$= x^2(x^2 - x - 6)$

$= x^2(x-3)(x+2)$

STEP2 Find x-intercept : Solve $x^4 - x^3 - 6x^2 = 0$

$\Rightarrow x^2(x-3)(x+2) = 0$

$\downarrow$ Z.F.T.

$x=0$ or $x-3=0$ or $x+2=0$

$\downarrow$

$x=0, x=3, x=-2$

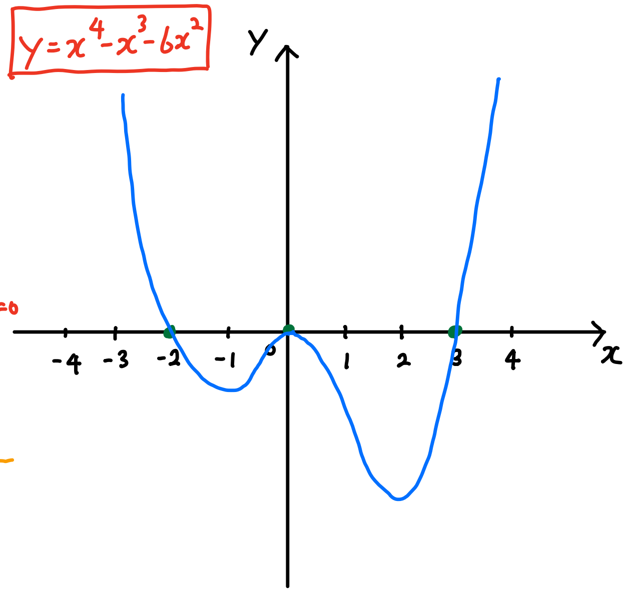
$\downarrow$

$(0,0), (3,0), (-2,0)$

y-intercept : $(0, 0)$

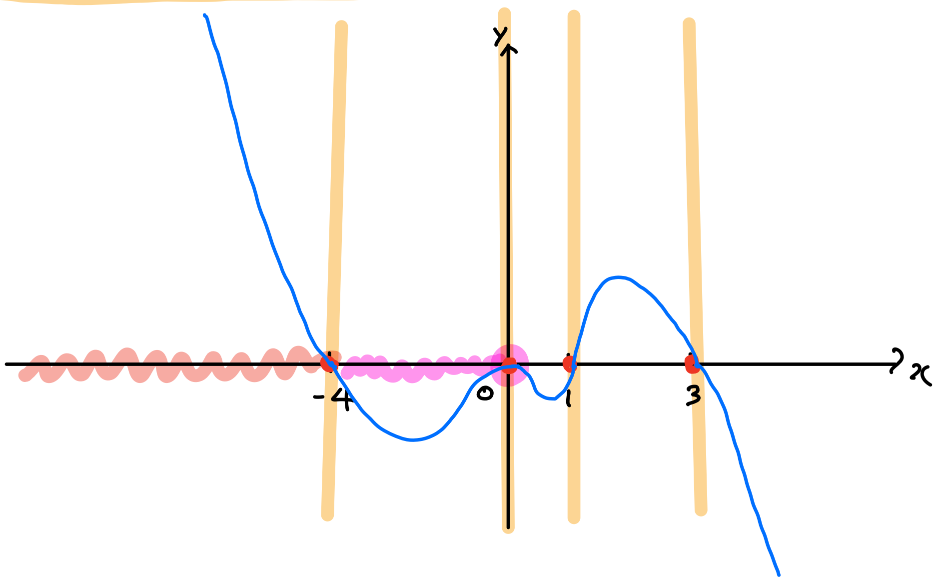
	$(-\infty,-2)$	-2	$(-2,0)$	0	$(0,3)$	3	$(3,\infty)$
$f(x)=x^2(x+3)(x-3)$	⊕		⊖		⊖		⊕
x^2 (it is x^2, NOT x.)	+		+	0	+		+
$x-3$	−		−		−	0	+
$x+2$	−	0	+		+		+

$y = x^4 - x^3 - 6x^2$



Ex Given the sign diagram below, sketch a possible graph of the polynomial f.

	$(-\infty,-4)$	-4	$(-4,0)$	0	$(0,1)$	1	$(1,3)$	3	$(3,\infty)$
Sign of $f(x)$	$+$		$-$		$-$		$+$		$-$



Section 4.2 Properties of Division.

Def Given two polynomials $f(x)$ and $g(x)$,

$f(x)$ is divisible by $g(x)$ if $g(x)$ is a factor of $f(x)$.

it means there exists a third polynomial $h(x)$ such that $\boxed{f(x) = g(x) \cdot h(x)}$

Ex x^2+5x+4 has four factors : 1, $(x+1)$, $(x+4)$, $(x+1)(x+4)$

" $(x+1)(x+4)$.

Hence x^2+5x+4 is divisible by 1, $(x+1)$, $(x+4)$, $(x+1)(x+4)$

When $f(x)$ is not divisible by $g(x)$, we can still divide $f(x)$ by $g(x)$ using "long division".

Recall : long division for positive integers.

$4 \cdot ? < 13$

$$\begin{array}{r} 3 \\ 4\,\overline{)\,13} \\ \underline{12} \\ 1 \end{array}$$

$$13 = 4 \cdot \underset{\text{quotient}}{3} + \underset{\text{remainder}}{1}$$

Ex

$$\begin{array}{rl} & 2096 \\ 11 & \overline{)\,23057} \\ & \underline{22\downarrow\downarrow\ \ } \\ & 105\ \ \\ & \underline{99\ \ } \\ & 67 \\ & \underline{66} \\ & 1 \end{array}$$

$\longrightarrow 23057 = 11 \times \underset{\text{quotient}}{\underline{2096}} + \underset{\text{remainder.}}{\underline{1}}$

Long division for the polynomials is similar.

<u>Ex</u> $x^2-2x+2\,\overline{\big)\,2x^5-3x^3+5x^2+1}$

$x^2\cdot 2x^3 = 2x^5$ $\qquad\Downarrow$

quotient.

$$
\begin{array}{r l}
 & \phantom{2x^5+{}}2x^3+4x^2+x-1 \\
x^2-2x+2 & \overline{\big)\,2x^5+0\cdot x^4-3x^3+5x^2+0\cdot x+1} \\
 & \underline{2x^5-4x^4+4x^3} \\
 & \phantom{2x^5-{}}4x^4-7x^3+5x^2 \\
 & \phantom{2x^5-{}}\underline{4x^4-8x^3+8x^2} \\
 & \phantom{2x^5-4x^4+{}}x^3-3x^2+0\cdot x \\
 & \phantom{2x^5-4x^4+{}}\underline{x^3-2x^2+2x} \\
 & \phantom{2x^5-4x^4+x^3{}}-x^2-2x+1 \\
 & \phantom{2x^5-4x^4+x^3{}}\underline{-x^2+2x-2} \\
 & \phantom{2x^5-4x^4+x^3-x^2{}}-4x+3
\end{array}
$$

remainder.

$2x^3(x^2-2x+2)=2x^5-4x^4+4x^3$

$2x^5+0\cdot x^4-3x^3-(2x^5-4x^4+4x^3)$

$=4x^4-7x^3$

$4x^2(x^2-2x+2)=4x^4-8x^3+8x^2$

$(4x^4-7x^3+5x^2)-(4x^4-8x^3+8x^2)$

$=x^3-3x^2$

$$\Rightarrow\ 2x^5-3x^3+5x^2+1 = (x^2-2x+2)(2x^3+4x^2+x-1)+(-4x+3)$$

$x^2-2x+2 \qquad\qquad x^2-2x+2$

$$= \frac{\cancel{(x^2-2x+2)}(2x^3+4x^2+x-1)}{\cancel{(x^2-2x+2)}} + \frac{(-4x+3)}{x^2-2x+2}$$

$$\frac{2x^5-3x^3+5x^2+1}{x^2-2x+2} = (2x^3+4x^2+x-1) + \frac{(-4x+3)}{x^2-2x+2}$$

Division Algorithm for Polynomials.

If $f(x)$ and $p(x)$ are polynomials and if $p(x) \neq 0$, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = p(x) \cdot q(x) + r(x)$$

where either $r(x) = 0$ or the degree of $r(x)$ is less than the degree of $p(x)$.