Section 3.7. Continued

HW5 ~~was due this Friday at 1pm.~~ is due this Saturday at 11:59 p.m.

What can we say about the domain of the composite function?

Domain of $f \circ g$ ($g \circ f$) is

$(f \circ g)(x) = f(g(x))$

x should be in $D(g)$.

$g(x)$ should be in $D(f)$.

$D(g) - \{$the set of all x such that $g(x)$ is not in $D(f)\}$.

$D(f) - \{$ —— " —— $f(x)$ is not in $D(g)\}$.

Ex Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{x}$.

Domain of $f = (-\infty, \infty)$

Domain of $g = [0, \infty)$

1) Domain of $f \circ g$?

$D(g) - \{$the set of all x such that $g(x)$ is not in $D(f)\}$

$\sqrt{x}$ is not in $(-\infty, \infty)$

$\sqrt{x}$: a real number $\Rightarrow \sqrt{x}$ is always in $(-\infty, \infty)$.

$[0, \infty) - \{$ Nothing $\}$.

$\Rightarrow \boxed{[0, \infty)}$

2) Domain of $g \circ f$?

$D(f) - \{$ —— " —— $f(x)$ is not in $D(g)\}$

$x^2 - 4$ is not in $\boxed{[0, \infty)}$

negative.

When $x^2 - 4 < 0$?

$x^2 - 2^2 < 0$

$(x+2)(x-2) < 0$.

$\boxed{-2 < x < 2}$: $(-2, 2)$

$(-\infty, \infty) - (-2, 2)$

$\Rightarrow \boxed{(-\infty, -2] \cup [2, \infty)}$.

Domain of f. Domain of g

No 6!

Ex Two functions f and g are given as follows:

x	2	3	4	5
$f(x)$	3	5	2	4

$f(4)=2$

Range f.

x	2	3	4	5
$g(x)$	4	1	3	6

Range of g.

Find $(f\circ g)(2)$, $(g\circ f)(3)$, $(f\circ f)(4)$, and $(g\circ g)(5)$.

$$(f\circ g)(2) = f(\underbrace{g(2)}_{4}) = f(4) = \boxed{2}.$$

$$(g\circ f)(3) = g(\underbrace{f(3)}_{5}) = g(5) = \boxed{6}.$$

$$(f\circ f)(4) = f(\underbrace{f(4)}_{2}) = f(2) = \boxed{3}.$$

$$(g\circ g)(5) = g(\underbrace{g(5)}_{6}) = g(6) = \boxed{\text{undefined}}$$ because 6 is not in the domain of g.

Finding a composite function form

: Express the given function as a composition of two functions

$y = h(x) \Rightarrow$ find $y = f(u)$ and $u = g(x)$ such that $h(x) = f(g(x)) = (f\circ g)(x)$

Ex Express 1) $y = (3x+1)^5$ as a composition of two functions.

2) $y = \sqrt[3]{2x^2+5}$

1) $y=(3x+1)^5$: Let us call $u = 3x+1$. Then $y = (3x+1)^5 = u^5$ and $u = 3x+1$.

Now, let $f(u) = u^5$ and $g(x) = 3x+1$.

Then $(f\circ g)(x) = f(g(x)) = g(x)^5 = (3x+1)^5$: ← it is the given equation!

2) $y = \sqrt[3]{2x^2+5}$: Let us call $u = 2x^2+5$. Then $y = \sqrt[3]{2x^2+5} = \sqrt[3]{u}$ and $u = 2x^2+5$.

Now, let $f(u) = \sqrt[3]{u}$ and $g(x) = 2x^2+5$.

Then $(f\circ g)(x) = f(g(x)) = \sqrt[3]{g(x)} = \sqrt[3]{2x^2+5}$: ← it is the given equation!

Chapter 4. Polynomial and Rational Functions.

$f(x) = a_n x^n + \cdots + a_1 x + a_0$.

ex) $f(x) = \dfrac{3x^2 - 5x + 1}{2x + 7}$

Section 4.1. Polynomial Functions of Degree Greater Than 2.

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$.

We have studied polynomial functions of degree 0, 1, and 2.

We first study $f(x) = x^n$. Ex $f(x) = x^3$, $f(x) = x^4$, ...

1) When n is odd : 1) $f(x) = x^n$ is an odd function. $f(-x) = -f(x)$

Ex) $n = 3$, $f(x) = x^3$
$f(-x) = (-x)^3 = -x^3 = -f(x)$

2) $f(x) = x^n$ is increasing on $\mathbb{R} = (-\infty, \infty)$

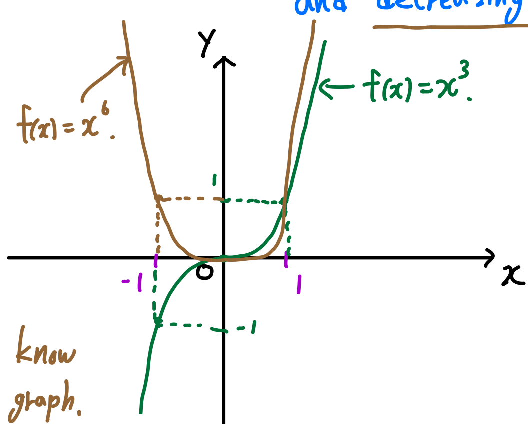
2) When n is even : 1) $f(x) = x^n$ is an even function. $f(-x) = f(x)$

Ex) $n = 6$, $f(x) = x^6$
$f(-x) = (-x)^6 = x^6$
$= f(x)$

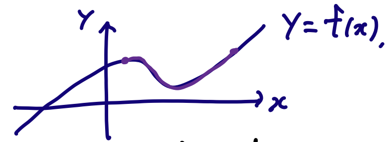
2) $f(x) = x^n$ is increasing on $[0, \infty)$, and decreasing on $(-\infty, 0]$

Ex ① $f(x) = x^3$

② $f(x) = x^6$



$f(x) = ax^n$: we also know about its graph.

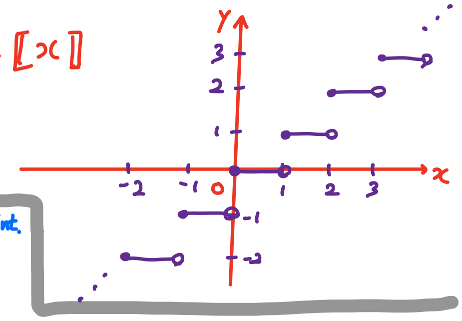


Continuous functions : Functions whose graphs can be drawn without any break.

Ex Every polynomial function.

Non Ex Greatest integer function

$Y = [\![x]\!]$

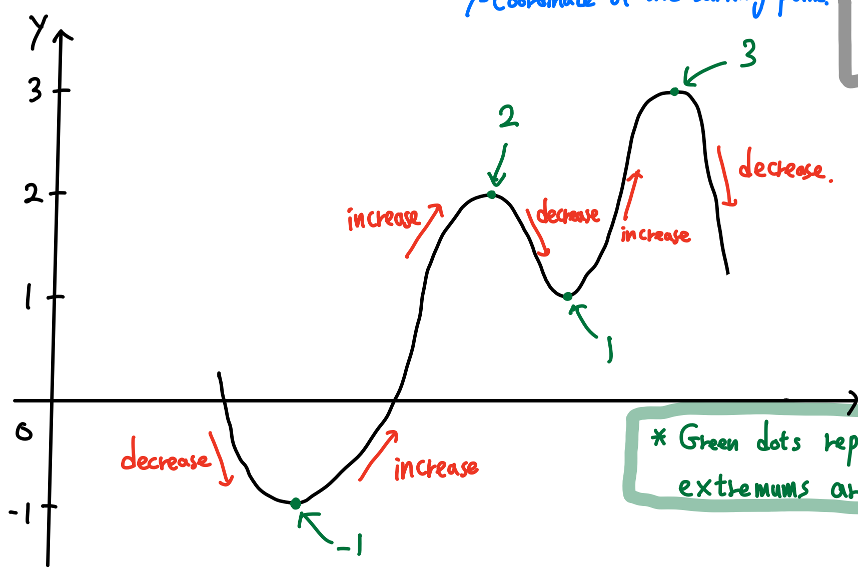


increase → decrease

decrease → increase.

Turning points and extremum

y-coordinate of the turning point



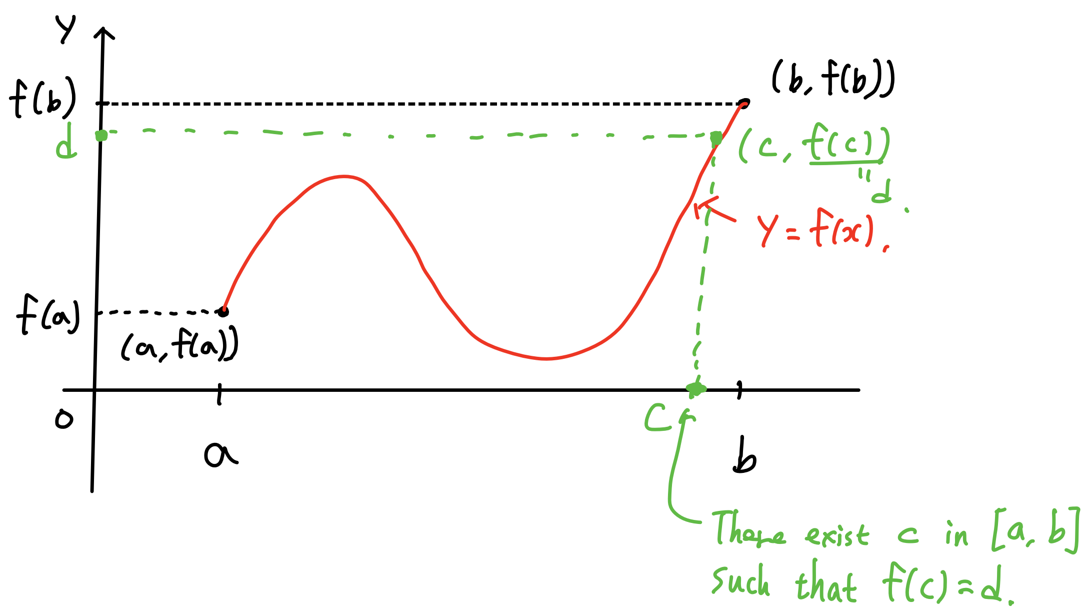
* Green dots represents turning points, and the extremums are denoted.

Intermediate Value Theorem (Version 1)

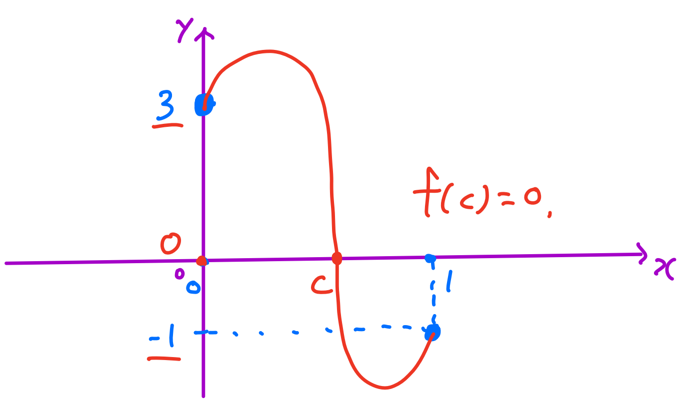
If f is a continuous function and $f(a) \neq f(b)$ for $a < b$, then f takes on every value between $f(a)$ and $f(b)$ in the interval $[a, b]$.

Intermediate Value Theorem (Version 2)

If f is a continuous function and $f(a) \neq f(b)$ for $a<b$, then for any d between $f(a)$ and $f(b)$, there exists at least one c in the interval $[a, b]$ such that $f(c)=d$.



Ex Show that $f(x) = 2x^4 - 3x^3 + x^2 - 4x + 3$ has a zero between 0 and 1.



$f(0) = 3$, $f(1) = 2-3+1-4+3 = -1$.

$f(x)$ is continuous, $f(0) = 3 \neq -1 = f(1)$

and 0 is in between -1 and 3.

Hence, by Intermediate Value Theorem, there exists c in $[0, 1]$ such that $f(c) = 0$: This c is a zero between 0 and 1!