Horizontal Stretching / Horizontal Compressing / Reflection through the y-axis.

Given a graph of $y=f(x)$ and for any positive number c, the graph of $y=f(cx)$ is obtained from the graph of $y=f(x)$ by { Compressing (if $c>1$) / stretching (if $0<c<1$) } the graph of $y=f(x)$ horizontally by a factor $\frac{1}{c}$.

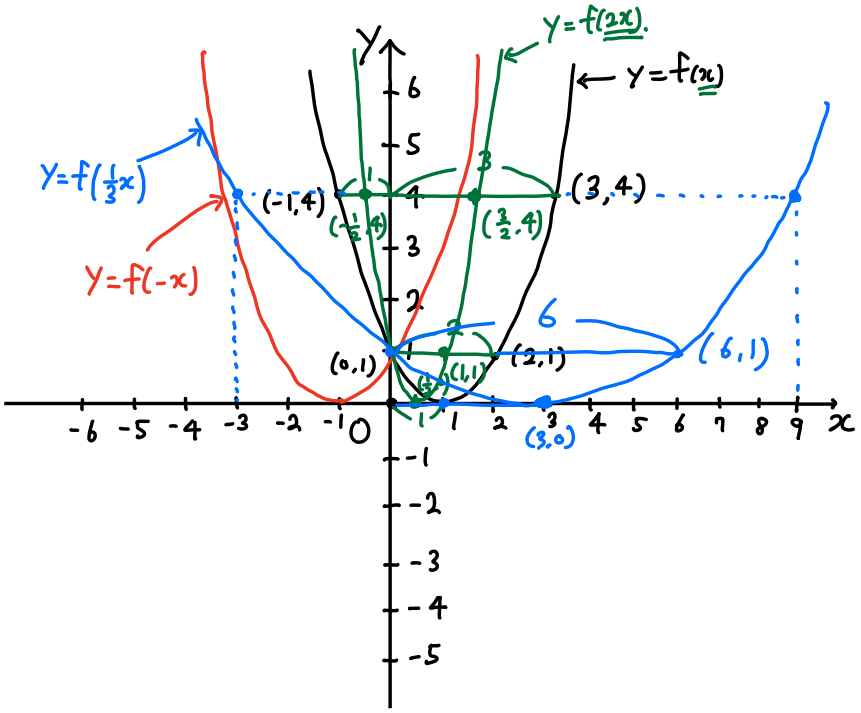
Given a graph of $y=f(x)$, the graph of $y=f(-x)$ is obtained by reflecting the graph of $y=f(x)$ through the y-axis.

Ex $f(x)=(x-1)^2$

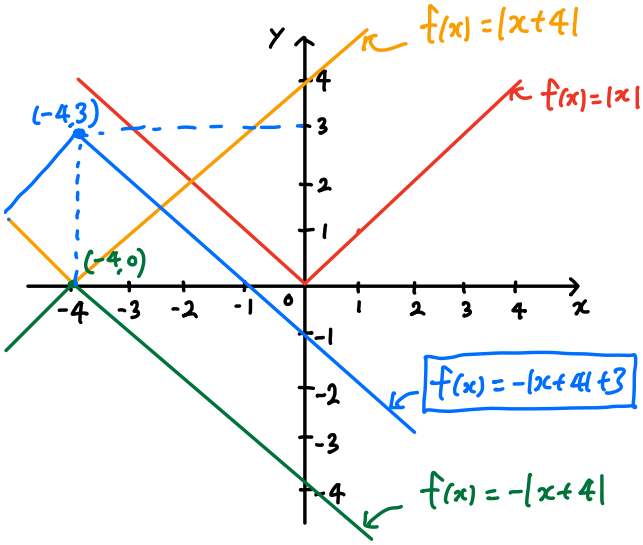
$f(2x)=(2x-1)^2$

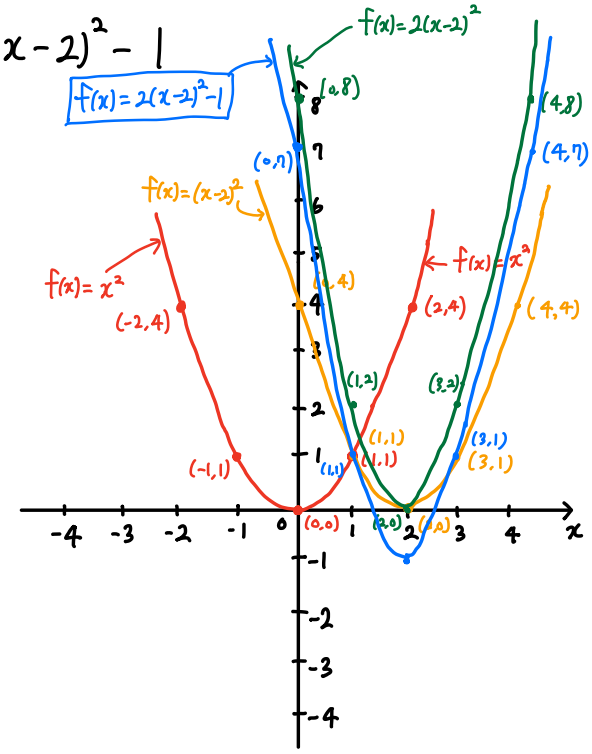
$f(\frac{1}{3}x)=(\frac{1}{3}x-1)^2$

$f(-x)=(-x-1)^2$



Find the graph of $\begin{cases} f(x) = -|x+4|+3 \\ f(x) = 2(x-2)^2 - 1 \end{cases}$





$f(x) = -|x+4|+3$

$f(x) = |x|$

↓ 4 unit to the left.

$f(x) = |x+4|$

↓ reflect through x-axis

$f(x) = -|x+4|$

↓ 3 units up

$f(x) = -|x+4|+3$

$f(x) = 2(x-2)^2 - 1$

$f(x) = x^2$

↓ 2 unit to the right

$f(x) = (x-2)^2$

↓ stretch vertically by factor 2

$f(x) = 2(x-2)^2$

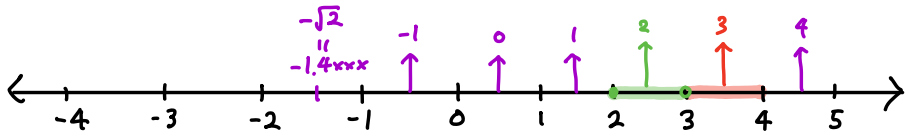
↓ 1 unit down

$f(x) = 2(x-2)^2 - 1$

The greatest integer function $f(x) = [\![x]\!]$

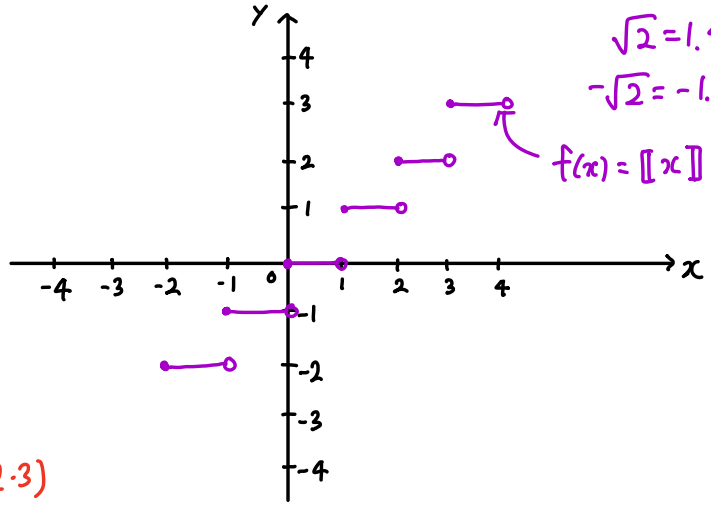
For any real number x, $[\![x]\!] = n$ where n is the greatest integer such that $n \le x$.

$[\![3]\!] = n$ $\qquad$ $n \le 3$



Ex ① $[\![3]\!] = 3$ $\qquad$ ③ $[\![-2]\!] = -2$ $\qquad$ ⑤ $[\![\pi]\!] = 3$

② $[\![0]\!] = 0$ $\qquad$ ④ $[\![1.7]\!] = 1$ $\qquad$ ⑥ $[\![-\sqrt{2}]\!] = -2$.



$\sqrt{2} = 1.41\times\times\times$

$-\sqrt{2} = -1.41\times\times\times$

$2\cdot(3)$ $\qquad$ $(2\cdot3)$

* $2(x)$ vs $(2x)$ $\quad$ $2[x]$ vs $[2x]$: They are the same

* $f(x) = [\![2x]\!]$ vs $f(x) = 2[\![x]\!]$: They are different!

Ex $x = 1.5$: $[\![2\cdot1.5]\!] = [\![3]\!] = 3$
$2\cdot[\![1.5]\!] = 2\cdot1 = 2$ } NOT the same!

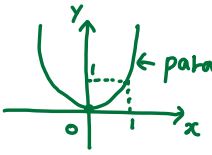
Section 3.6. Quadratic Functions.

Quadratic equation : $ax^2+bx+c=0$ with $a\neq 0$, $a, b,$ and c are real numbers.

Quadratic function : $f(x)=ax^2+bx+c$ with $a\neq 0$, $a, b,$ and c are real numbers.

↖ its graph is the graph of $y=ax^2+bx+c$.

We have seen the graph of $f(x)=x^2$.

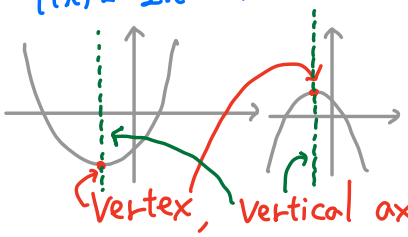


$f(x)=x^2$
↓
$f(x)=ax^2$

From Section 3.5, we also know how to draw $f(x)=ax^2+c$.

Ex Draw $f(x)=-2x^2+4$

$f(x)=x^2$
↓ vertical stretching
$f(x)=2x^2$
↓ reflection through x-axis.
$f(x)=-2x^2$
↓
$f(x)=-2x^2+4$



$f(x)=-2x^2+4$