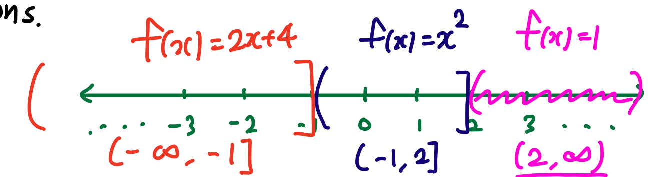
Section 3.5 Continued.

* Two Homework 4 are due tomorrow at 1pm!

Piecewise-defined functions.



Domain of the functions is split into several pieces, and the function is defined differently on each piece.

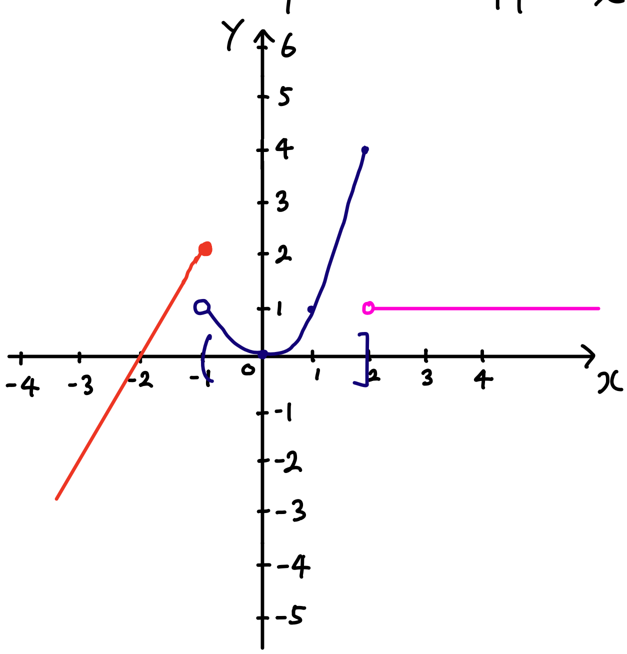
Ex $f(x) = \begin{cases} 2x+4 & \text{if } x \le -1 \\ x^2 & \text{if } -1 < x \le 2 \\ 1 & \text{if } x > 2 \end{cases}$

$-3 \le -1$

$f(-3) = 2\cdot(-3)+4$
$= -6+4$
$= -2.$

$f(3) = 1$

$3 > 2$



Graph of an equation containing an absolute function.

$$\left(|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases} \right)$$

Ex ⓪ $y = |x|$: $y = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$\boxed{y = x}$

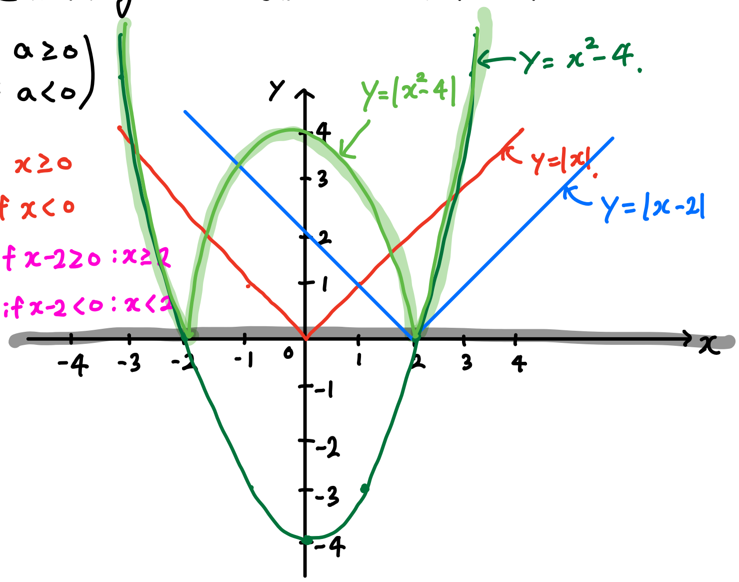
① $y = |x-2|$: $y = \begin{cases} x-2 & \text{if } x-2 \geq 0 : x \geq 2 \\ -(x-2) & \text{if } x-2 < 0 : x < 2 \end{cases}$

$y = x-2$

② $y = |x^2 - 4|$

$y = x^2 - 4.$

x	y
-3	$(-3)^2-4 = 9-4 = 5$
-2	$(-2)^2-4 = 4-4 = 0$
-1	-3
0	-4
1	-3
2	0
3	5



Vertical Shift / Horizontal Shift of the graphs.

Given a graph of $y=f(x)$ and for any positive number c,

the graph of $y=f(x)+c$ is obtained from the graph of $y=f(x)$ by $\uparrow$ c units.

the graph of $y=f(x)-c$ is obtained from the graph of $y=f(x)$ by $\downarrow$ c units.

the graph of $y=f(x+c)$ is obtained from the graph of $y=f(x)$ by $\longleftarrow$ c units.

we replace x by $x+c$.

the graph of $y=f(x-c)$ is obtained from the graph of $y=f(x)$ by $\longrightarrow$ c units.

replace x by $x-c$.

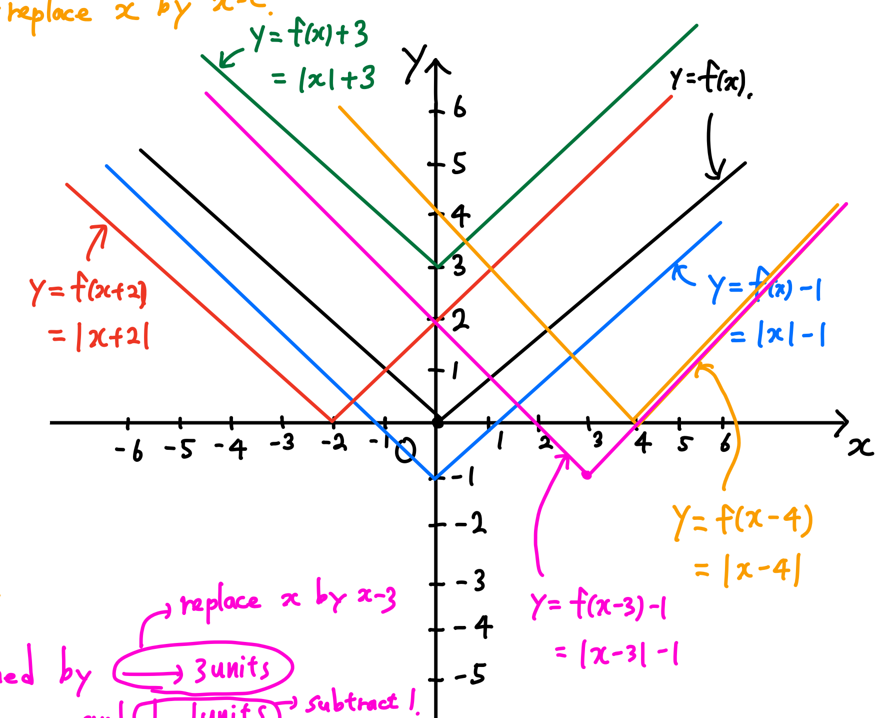
<u>Ex</u> $f(x)=|x|$

$f(x)+3=|x|+3$

$f(x)-1=|x|-1$

$f(x+2)=|x+2|$

$f(x-4)=|x-4|$

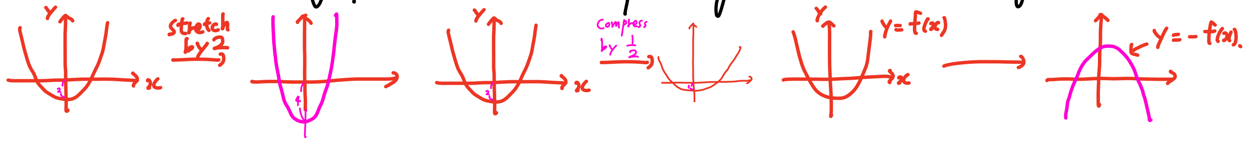


Pink graph is obtained by $\longrightarrow$ 3 units (replace x by $x-3$)

and $\downarrow$ 1 units (subtract 1)

$|x| \longrightarrow |x-3|-1$

Vetical Stretching / Vertical Compressing / Reflection through the x-axis.



Given a graph of $y=f(x)$ and for any positive number c,

the graph of $y=c\cdot f(x)$ is obtained from the graph of $y=f(x)$

by $\begin{cases} \text{stretching (if } c>1) \text{ the graph of } y=f(x) \text{ vertically by a factor } c. \\ \text{Compressing (if } 0<c<1) \end{cases}$

Given a graph of $y=f(x)$, the graph of $y=-f(x)$ is obtained

by reflecting the graph of $y=f(x)$ through the x-axis.

Ex $f(x)=x^2-1$

$-f(x) = -(x^2-1) = -x^2+1$

Stretch the function by 3 (vertically).

$\Rightarrow 3f(x) = 3(x^2-1) = \boxed{3x^2-3}$

Compress the function by $\frac{1}{2}$

$\Rightarrow \frac{1}{2}f(x) = \frac{1}{2}(x^2-1) = \boxed{\frac{1}{2}x^2-\frac{1}{2}}$