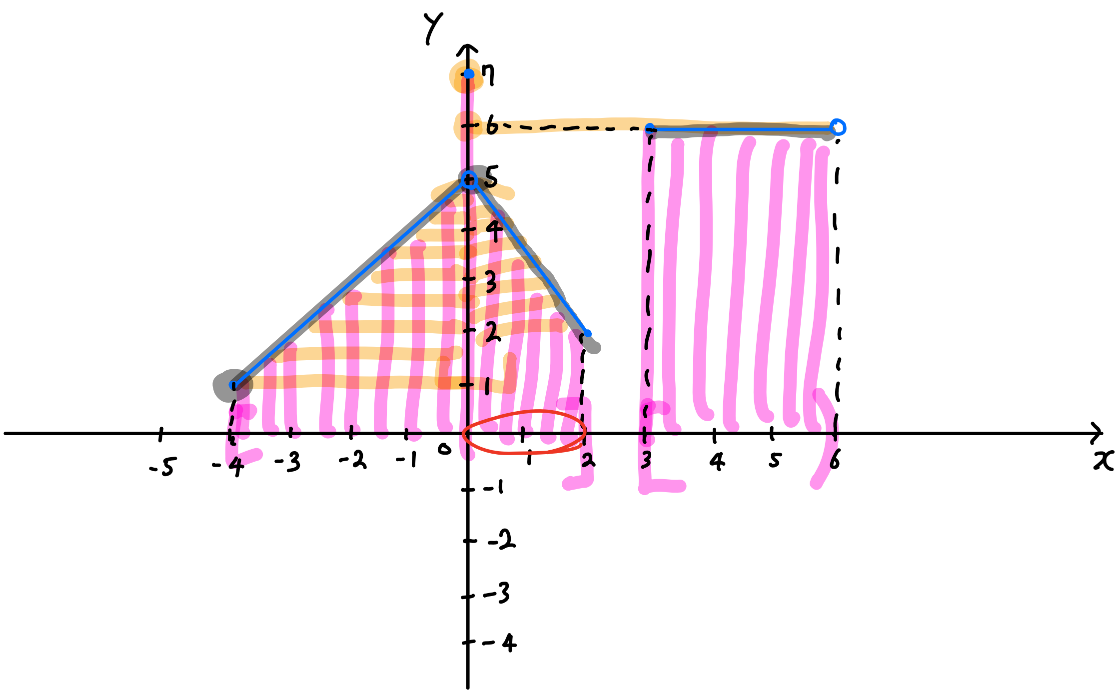
Section 3.4. Continued

General Form of the graph of a linear line
: $\boxed{ax+by=c}$, NOT $ax+by+c=0$.
When you do Online Homework 4, the general form of the equation should satisfy two additional conditions :
① $a>0$, ② a, b, and c do not have the common factor.
Ex $\boxed{y=\frac{3}{2}x+1} \xrightarrow{\times 2} 2y=3x+2 \xrightarrow{-3x} -3x+2y=2 \xrightarrow{\times(-1)} \boxed{3x-2y=-2}$
Please check the Canvas today (Practice Exam)
Exam 1 : 1) No Calculator
2) Bring your ID.

Ex The following is the graph of function f.



① Find the domain and the range of f.

$[-4, 2] \cup [3, 6)$

$[1, 5) \cup \{6\} \cup \{7\} = [1, 5) \cup \{6, 7\}$
: an interval $[1,5)$ and two points 6 and 7.

② Find the intervals on which f is increasing or decreasing.

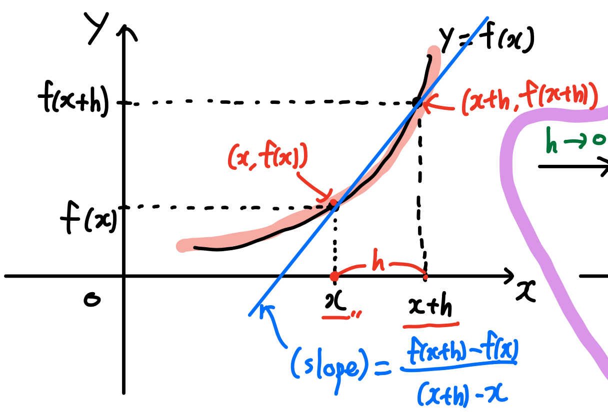
$\nearrow$ $\Downarrow$ $[-4, 0]$

$\searrow$ $\Downarrow$ $[0, 2]$

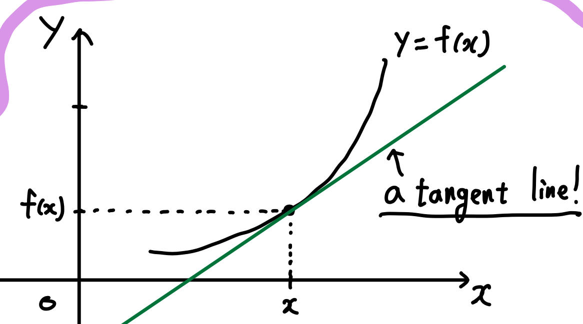
Given a function $f(x)$, we should be able to simplify the expression $\dfrac{f(x+h)-f(x)}{h} = \dfrac{f(x+h)-f(x)}{(x+h)-x}$ $\dfrac{f(x)-f(a)}{x-a}$

called difference quotient.

M211 material



$h \to 0$



As $h \to 0$, the difference quotient approaches to the slope of the tangent line!

$f'(x) = 4x - 3$ (M211)

Ex Simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ using the function $f(x) = 2x^2 - 3x + 1$ $\Rightarrow f(x+h) = 2(x+h)^2 - 3(x+h) + 1$

$$\frac{f(x+h)-f(x)}{h} = \frac{\left(2(x+h)^2 - 3(x+h)+1\right) - \left(2x^2-3x+1\right)}{h}$$

$\boxed{(x+h)^2 = x^2+2xh+h^2}$

$$= \frac{\left(2(x^2+2xh+h^2) - 3(x+h)+1\right) - \left(2x^2-3x+1\right)}{h}$$

$$= \frac{(2x^2+4xh+2h^2-3x-3h+1)-(2x^2-3x+1)}{h}$$

$$= \frac{4xh+2h^2-3h}{h}$$

$$= \frac{h(4x+2h-3)}{h}$$

$= \boxed{4x+2h-3}$

if $h \to 0$

M211

$4x-3$

Linear equation in x : $ax+b=0$

Linear function in x : $f(x) = \underline{ax+b}$ (a, b can be any real number)

* When $a=1$ and $b=0$, we get $f(x)=x$

it is called identity function.

Its graph is the graph of $y=ax+b$

($f(x)=ax+b$ with $a=\frac{3}{2}$ and $b=-2$)

Ex Let $f(x)=\frac{3}{2}x-2$

(a) Sketch the graph of f. $\cancel{f(x)}\ Y = \frac{3}{2}x-2$: $\boxed{y=\frac{3}{2}x-2}$

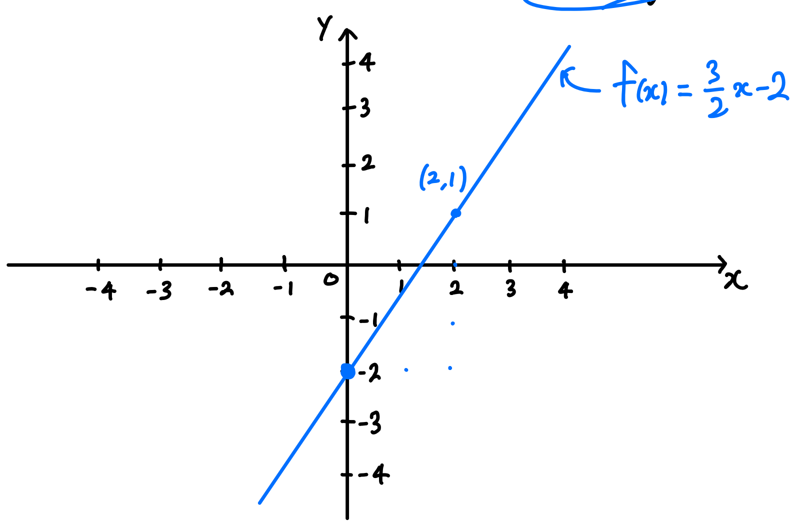
y-intercept,

slope.

(b) Find the domain and range of f.

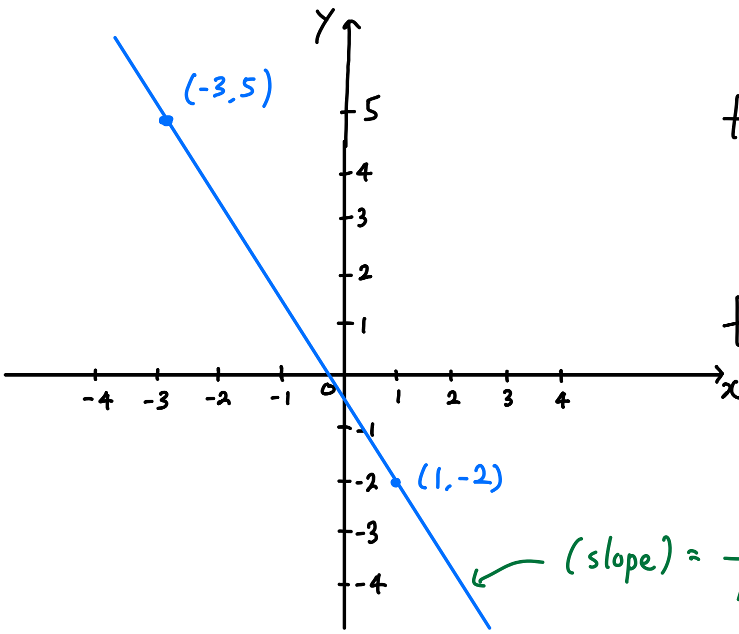
all real numbers, $\mathbb{R}$

(c) Determine where f is increasing or is decreasing.



$x=-3 \longrightarrow 5$: y-coordinate.

Ex If f is a linear function such that $f(-3)=5$ and $f(1)=-2$, find $f(x)$ where x is any real number.



$f(x)=ax+b$.

$f(-3)=5 \Rightarrow (-3,5)$ is on the graph of $f(x)$.

$f(1)=-2$, $\Rightarrow (1,-2)$ is on the graph of $f(x)$.

$$(\text{slope}) = \frac{-2-5}{1-(-3)} = \frac{-7}{4} = -\frac{7}{4}$$

A line passing through $(1,-2)$ whose slope is $-\frac{7}{4}$

$$\Rightarrow y-(-2) = -\frac{7}{4}(x-1). \quad y+2 = -\frac{7}{4}x+\frac{7}{4} \Rightarrow y = -\frac{7}{4}x-\frac{1}{4}$$

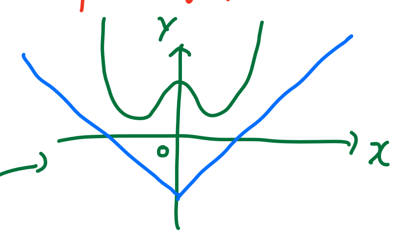
$-2 \qquad -2$

$$\Rightarrow \boxed{f(x) = -\frac{7}{4}x-\frac{1}{4}}$$

Section 3.5. Graphs of function.

Odd function / Even function

(Two examples of graphs of even function)



A function f is "even function" if its graph is symmetric with respect to the y-axis

How can we check it?

Domain of f

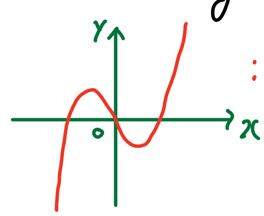
☆ A function f is "even function" if $\boxed{f(-x)=f(x)}$ for every $x \in D$

Ex ① $f(x)=3x^4-2x^2-5$ — same — it is even.
$f(-x)=3\cdot(-x)^4-2(-x)^2-5=3x^4-2x^2-5$

② $f(x)=|x|-2$ — same — it is even.
$f(-x)=|-x|-2=|x|-2$ (Recall $|-x|=|x|$)

A function f is "odd function" if its graph is symmetric with respect to the origin.

An example :



How can we check it?

Domain of f

☆ A function f is "odd function" if $\boxed{f(-x)=-f(x)}$ for every $x \in D$

Ex ① $f(x)=2x^3-4x$ → it is odd.
$f(-x)=2(-x)^3-4(-x)$
$=-2x^3+4x=-(\underbrace{2x^3-4x}_{f(x)})=-f(x)$

② $f(x)=x\cdot|x|$ → it is odd.
$f(-x)=-x\cdot|-x|=-\underbrace{x\cdot|x|}_{f(x)}=-f(x)$

Vertical Shift / Horizontal Shift of the graphs.

Given a graph of $y=f(x)$ and for any positive number c,

the graph of $y=f(x)+c$ is obtained from the graph of $y=f(x)$ by $\uparrow$ c units.

the graph of $y=f(x)-c$ is obtained from the graph of $y=f(x)$ by $\downarrow$ c units.

the graph of $y=f(x+c)$ is obtained from the graph of $y=f(x)$ by $\longleftarrow$ c units.

the graph of $y=f(x-c)$ is obtained from the graph of $y=f(x)$ by $\longrightarrow$ c units.

Ex $f(x)=|x|$