Section 3.4. Continued

Set : a collection of objects of some type

elements.

Ex $A = \{\text{apple, banana, watermelon}\}$

$B = \{1, 2, 3, 4, 5\} = \{x \mid x \text{ is a natural number less than } 6\}$

it consists of all natural numbers that are less than 6.

$C = \{2, 3, 5, 7\} = \{x \mid x \text{ is a prime number less than } 10\}$

Notation $3 \in C$ means 3 is an element of the set C.
$4 \notin C$ means 4 is NOT an element of the set C.

Ex Write a set that represents all real numbers such that the following expression makes sense.

① $\frac{1}{x}$: all real numbers different from zero. $= \mathbb{R} - \{0\}$ (the set of all real numbers.)

$\{x \mid x \text{ is a real number different from } 0\} = (-\infty, 0) \cup (0, \infty)$

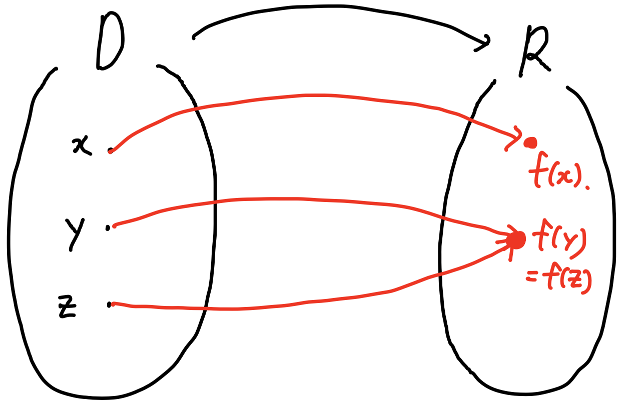
② $\frac{1}{x-2} + \sqrt{x-1}$: $\{x \mid x \text{ is a real number greater than or equal to } 1 \text{ and not equal to } 2\}$: $[1, 2) \cup (2, \infty)$

$x \neq 2$ $\quad$ $x \geq 1$

Suppose two sets D and R are given.

A function f from D to R is a correspondence that assign each element x of D exactly one element in R.

f : f maps D into R.

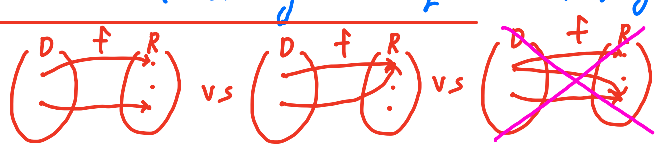


D : Domain of f.

R : Range of f.

* "Two functions f and g are equal" if $f(x)=g(x)$ for all x in D.

Ex

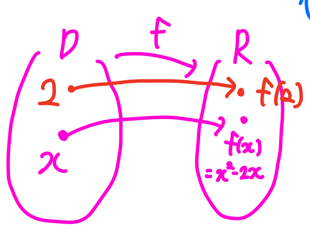


Not a function

Given a function f, finding a value $f(t)$ is straightforward.

Ex If $f(x)=x^2-2x$, what is $f(2)=2^2-2\cdot 2=4-4=\boxed{0}$,

(Domain) = $\mathbb{R}$



$f(-a)=(-a)^2-2(-a)$

$=\boxed{a^2+2a}$

Domain of the functions are usually not provided.

But, you should be able to find an implied domain by using the facts that

1) Denominator of Rational Expressions cannot be zero.

2) When n is even, we are only allowed to have non-negative numbers in $\sqrt[n]{}$

$\underline{\text{Ex}}$ Let $h(x) = \dfrac{1}{x-2} + \sqrt{x-1}$. $x-1 \geq 0$ $\boxed{x \geq 1}$

it cannot be zero $\Rightarrow \boxed{x \neq 2}$

1) Find the domain of h. $\{x \mid x \text{ is a real number} \geq 1 \text{ and } \neq 2\} = [1,2) \cup (2,\infty)$

2) Find $h(3)$, $h(-1)$ and $h(-t)$

$$h(3) = \frac{1}{3-2} + \sqrt{3-1} = \frac{1}{1} + \sqrt{2} = \boxed{1+\sqrt{2}}$$

$$h(-1) = \boxed{\text{DNE}}$$

Does Not Exist.

$$h(-t) = \frac{1}{-t-2} + \sqrt{-t-1} = \frac{1}{-(t+2)} + \sqrt{-t-1} = \boxed{-\frac{1}{t+2} + \sqrt{-t-1}}$$

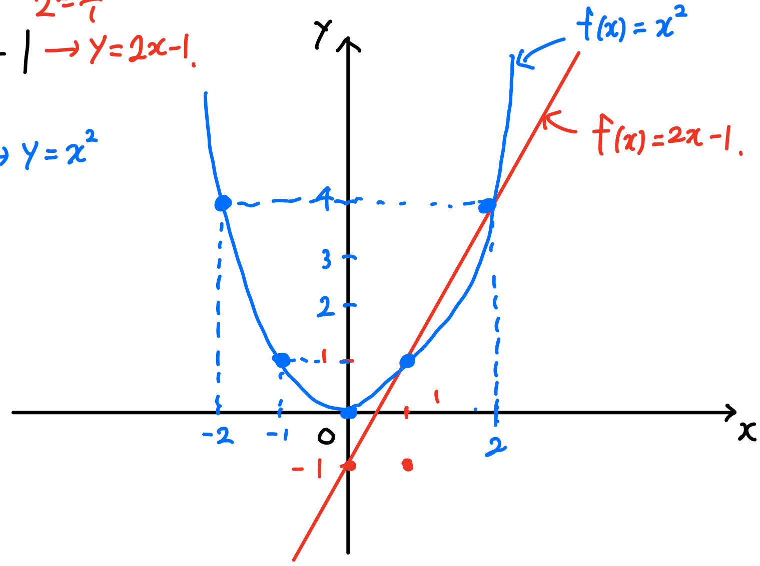
Given a function we consider its graph.

→ The graph of a function $f(x)$ is the graph of the equation $y = f(x)$.

$\underline{\text{Ex}}$ ① $f(x) = 2x - 1 \rightarrow y = 2x-1$. $2 = \frac{2}{1}$

② $f(x) = x^2 \rightarrow y = x^2$

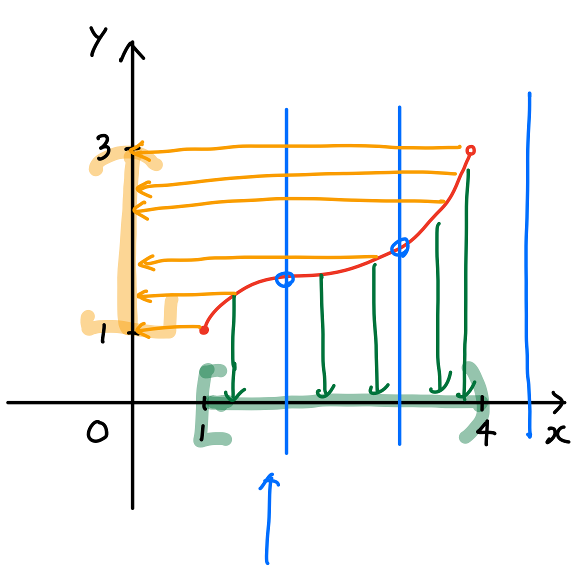
x	y
(-2,	4)
(-1,	1)
(0,	0)
(1,	1)
(2,	4)



Conversely, given a graph, we can easily check whether it is a graph of a function or not.

Vetical Line Test

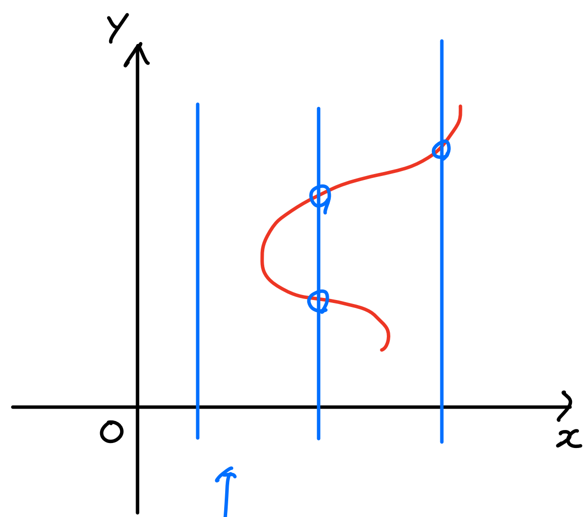
: The graph is the graph of a function if every vertical line intersects the graph in at most one point.



Graph of a function.

Domain : [1, 4)

Range : [1, 3)



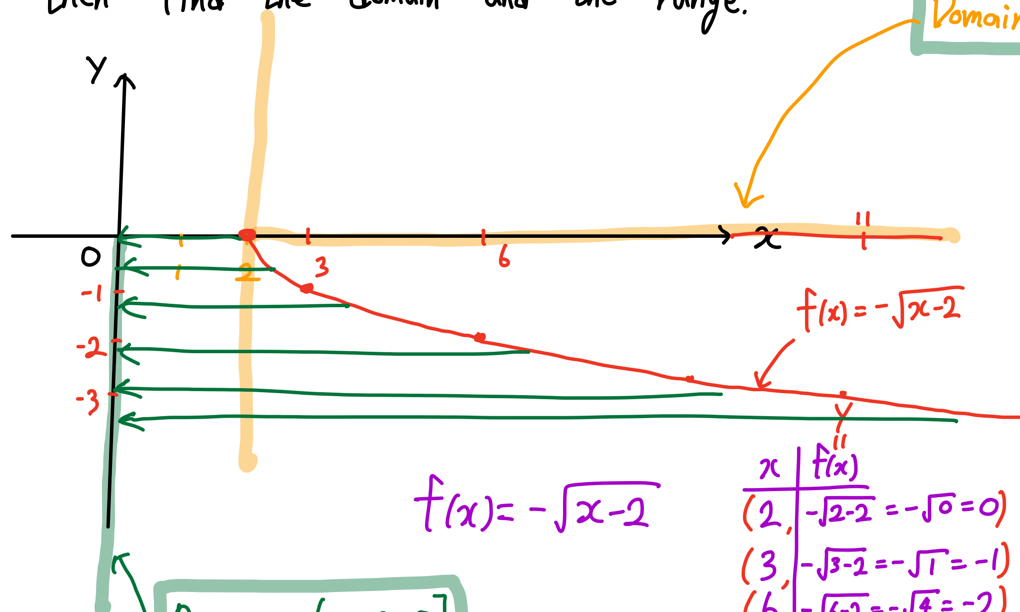
Not a graph of a function.

Ex Sketch the graph of $f(x) = -\sqrt{x-2}$ then find the domain and the range.

$x - 2 \geq 0$

$\boxed{x \geq 2}$

$\boxed{\text{Domain} = [2, \infty)}$



$\boxed{\text{Range} = (-\infty, 0]}$

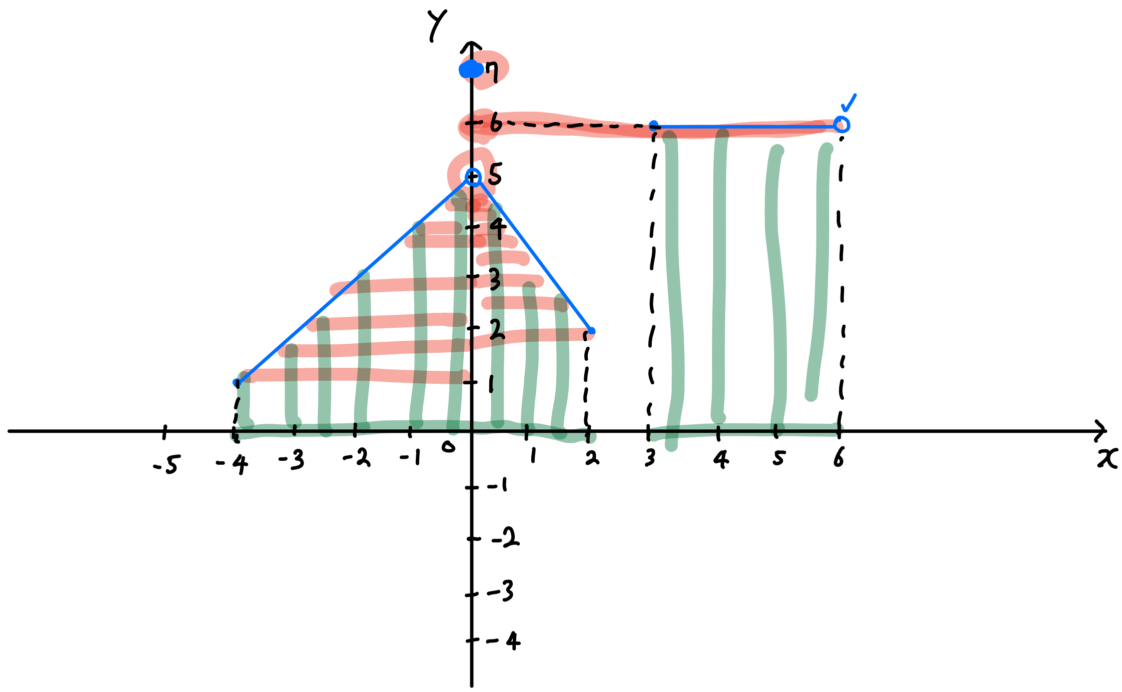
x	$f(x)$
$(2,$	$-\sqrt{2-2} = -\sqrt{0} = 0)$
$(3,$	$-\sqrt{3-2} = -\sqrt{1} = -1)$
$(6,$	$-\sqrt{6-2} = -\sqrt{4} = -2)$
$(11,$	$-\sqrt{11-2} = -\sqrt{9} = -3)$

The function f is increasing ↗ on an interval I if "$f(x_1) < f(x_2)$ whenever $x_1 < x_2$" or "the graph of f is rising".

The function f is decreasing ↘ on an interval I if "$f(x_1) > f(x_2)$ whenever $x_1 < x_2$" or "the graph of f is falling".

The function f is constant → on an interval I if "$f(x_1) = f(x_2)$ for every x_1, x_2" or "the graph of f is horizontal."

Ex The following is the graph of function f.



① Find the domain and the range of f.

$\boxed{[-4, 2] \cup [3, 6)}$

$[1, 5) \cup \{6\} \cup \{7\} = [1, 5) \cup \{6, 7\}$

: an interval $[1,5)$ and two points 6 and 7.

② Find the intervals on which f is increasing or decreasing.

We will do the second part next Monday.