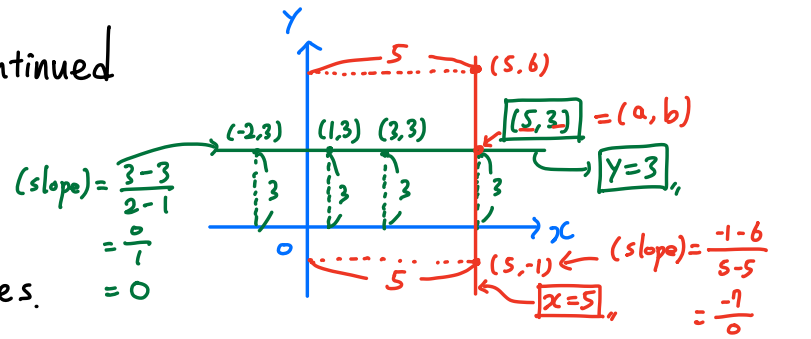
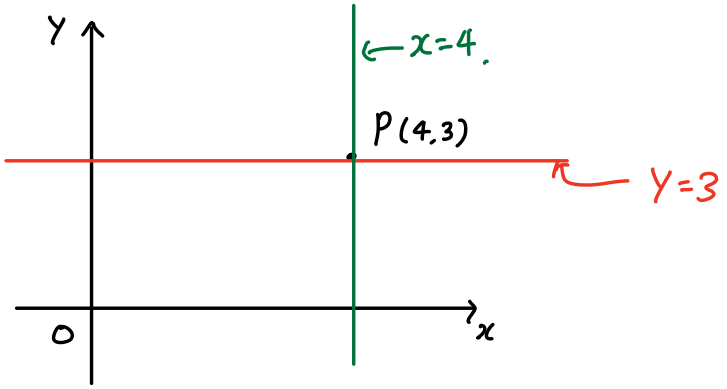
Section 3.3 Continued



Two special lines.

- Horizontal line through $P(a, b)$ is a graph of the equation "$y = b$". Its slope is 0
- Vertical line through $P(a, b)$ is a graph of the equation $x = a$. Its slope is undefined.



Point - Slope Form for the equation of a Line:

An equation for the line through the point (x_1, y_1) with slope m is

$$y - y_1 = m(x - x_1)$$

Ex Find an equation of the line through $C(3, 7)$ (x_1, y_1) and $D(-1, 1)$ (x_2, y_2).

$(3, 7)$ $m = \frac{3}{2}$

$y - 7 = \frac{3}{2}(x - 3)$

$y - 7 = \frac{3}{2}x - \frac{9}{2}$ $\xrightarrow{+7}$ $y = \frac{3}{2}x - \frac{9}{2} + \frac{14}{2}$

$y = \frac{3}{2}x + \frac{5}{2}$

$(\text{slope}) = \frac{3}{2}$ $\quad$ y-intercept is $(0, \frac{5}{2})$

$$(\text{slope}) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 7}{-1 - 3} = \frac{-6}{-4} = \frac{3}{2}$$

Slope-Intercept Form for the Equation of a Line:

The graph of $y = mx + b$ is a line having slope m and y-intercept b.

Replace $x = 0 \Rightarrow y = m \cdot 0 + b = b \Rightarrow (0, b)$ is y-intercept.

General Form for the Equation of a Line:

The graph of a linear equation $ax + by = c$ is a line, and conversely, every line is the graph of a linear equation.

$y = \frac{3}{2}x + \frac{5}{2} \xrightarrow{-y} \frac{3}{2}x - y + \frac{5}{2} = 0 \xrightarrow{\times 2} 3x - 2y + 5 = 0 \xrightarrow{-5} 3x - 2y = -5$

Ex Express the equation $5x - 3y = 9$ in slope-intercept form $y = mx + b$, and sketch the graph.

$5x - 3y = 9$

$\downarrow -5x$

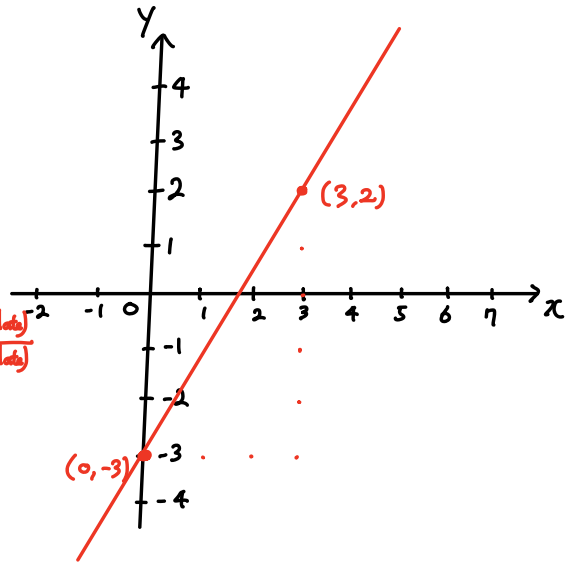
$-3y = -5x + 9$

$\downarrow \div(-3)$

$y = \frac{5}{3}x - 3$

$(\text{slope}) = \frac{5}{3} = \frac{(\text{difference of } y\text{-coordinates})}{(\text{difference of } x\text{-coordinates})}$

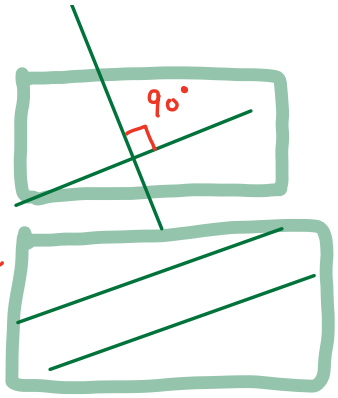
$(y\text{-intercept}) = (0, -3)$



Parallel lines and Perpendicular lines

↑ do not intersect

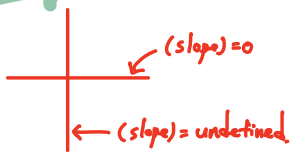
↑ intersect at a right angle

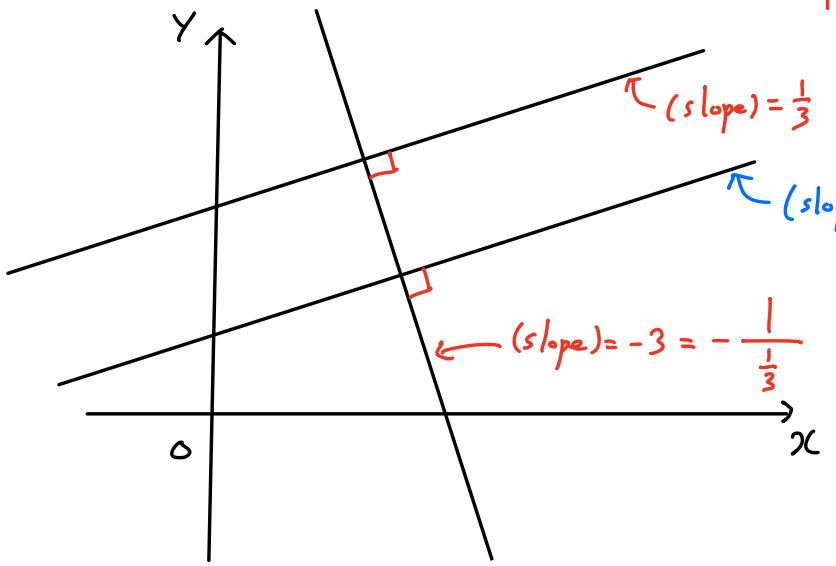


Useful facts about Parallel lines and Perpendicular lines.

① Two non vertical lines are parallel if and only if they have the same slope.

② Two lines with slope m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$.





Ex Find the slope-intercept form for the line through $(4,-3)$ that is ① parallel to the line $2x+5y=3$ ② perpendicular to the line $2x+5y=3$.

$2x+5y=3$

$\downarrow -2x$

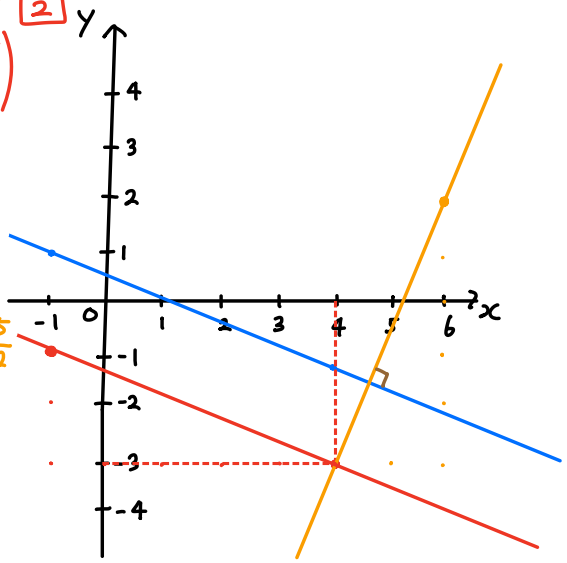
$5y=-2x+3$

$\downarrow \div 5$

$y=-\frac{2}{5}x+\frac{3}{5}$: (slope) $=-\frac{2}{5}$

(slope) of ① $=-\frac{2}{5}$

(slope) of ② $=\frac{5}{2}$ $\left(-\frac{1}{-\frac{2}{5}}=\frac{5}{2}\right)$



① (x_1,y_1) $(4,-3)$, m (slope) $=-\frac{2}{5}$

$y-(-3)=-\frac{2}{5}(x-4)$

$y+3=-\frac{2}{5}x+\frac{8}{5}$

$\downarrow -3$

$y=-\frac{2}{5}x+\frac{8}{5}-\frac{15}{5}$

$\Rightarrow \boxed{y=-\frac{2}{5}x-\frac{7}{5}}$

② $(4,-3)$, (slope) $=\frac{5}{2}$

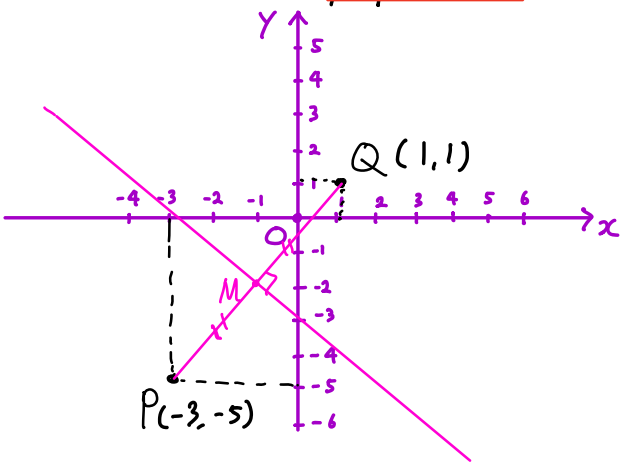
$y-(-3)=\frac{5}{2}(x-4)$

$y+3=\frac{5}{2}x-10$

$\downarrow -3$

$\boxed{y=\frac{5}{2}x-13}$

Ex Given $P(-3,-5)$ and $Q(1,1)$, find the equation of the perpendicular bisector ℓ of segment PQ.



STEP 1 Find the midpoint M of the segment PQ.

By the midpoint formula,

$\left(\frac{-3+1}{2},\frac{-5+1}{2}\right)=\left(\frac{-2}{2},\frac{-4}{2}\right)=(-1,-2)$

STEP 2 Find the slope of a line through P and Q.

By the slope formula,

$(\text{slope})=\frac{1-(-5)}{1-(-3)}=\frac{6}{4}=\frac{3}{2}$.

From STEP 1 and 2, we know that the perpendicular bisector ℓ passes through $(-1,-2)$ and its slope is $-\frac{2}{3}$.

Then, by the point-slope formula:

$y-(-2)=-\frac{2}{3}(x-(-1)) \Rightarrow y+2=-\frac{2}{3}x-\frac{2}{3} \Rightarrow \boxed{y=-\frac{2}{3}x-\frac{8}{3}}$

Section 3.4. Definition of Function

Set : a collection of objects of some type

elements.

Ex $A = \{ \text{apple}, \text{banana}, \text{watermelon} \}$

$B = \{ 1, 2, 3, 4, 5 \} = \{ x \mid x \text{ is a natural number less than } 6 \}$

it consists of all natural numbers that are less than 6.

$C = \{ 2, 3, 5, 7 \} = \{ x \mid x \text{ is a prime number less than } 10 \}$

set builder notation.

Notation $3 \in C$ means 3 is an element of the set C.

$4 \notin C$ means 4 is NOT an element of the set C.