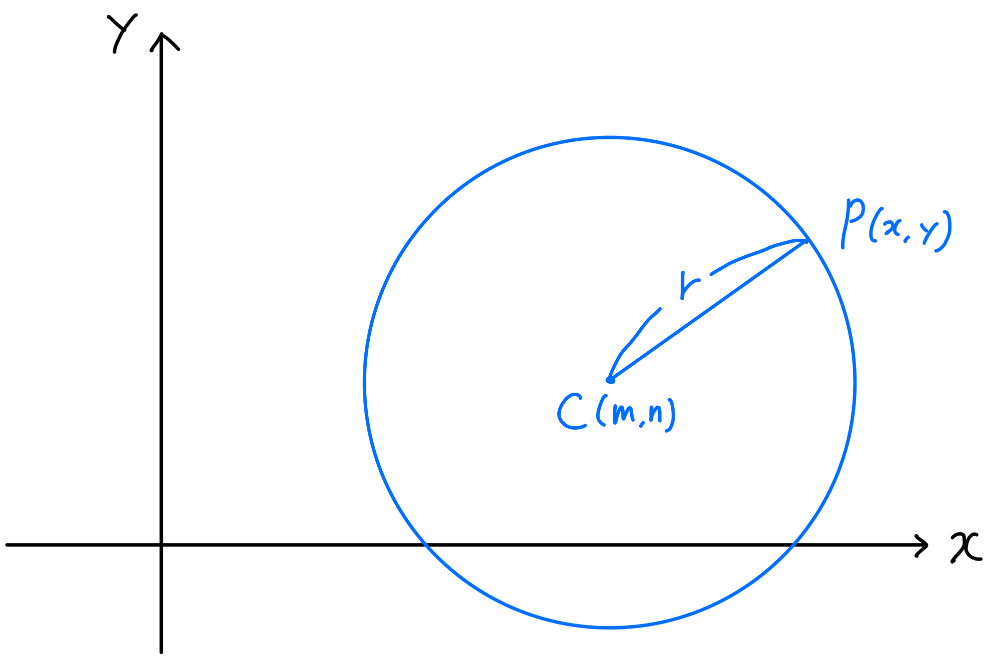
Section 3.2 Continued

- One of the exam problem will be a sentence problem
- Sections that we have covered and will cover until this Friday will be on the Exam I.

(Recall) Equation of a circle



By the distance formula: $\sqrt{(x-m)^2+(y-n)^2} = r$

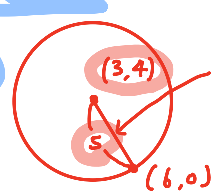
$$\boxed{(x-m)^2+(y-n)^2 = r^2}$$

: Standard equation of a circle

standard equation of a circle with $\boxed{(\text{center}) = (m, n)}$ and $\boxed{(\text{radius}) = r}$

<u>Ex</u> Find an equation of a circle that has center $(3, 4)$ and contains the point $(6, 0)$

$\Rightarrow \boxed{(x-3)^2+(y-4)^2 = 25}$



this length is the radius!

By distance formula, it is $\sqrt{(6-3)^2+(0-4)^2} = \sqrt{3^2+(-4)^2} = \sqrt{9+16} = \sqrt{25} = 5.$

Ex Find the center and the radius of the circle with equation $-4x^2-4y^2+8x-16y=-32$

$\downarrow \div(-4)$

$x^2+y^2-2x+4y=8$

$x^2-2x+a=(x+b)^2$
$=x^2+2bx+b^2$

$2b=-2,\ \boxed{b=-1}$

$a=b^2=(-1)^2=1$

$x^2-2x+1=(x-1)^2$

$y^2+4y+a=(y+b)^2$
$=y^2+2by+b^2$

$4=2b,\ b=2$

$a=b^2=4$

$y^2+4y+4=(y+2)^2$

$(x^2-2x)+(y^2+4y)=8$

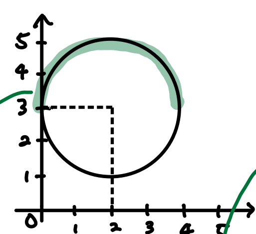
$(x^2-2x+1)+(y^2+4y)=8+1$

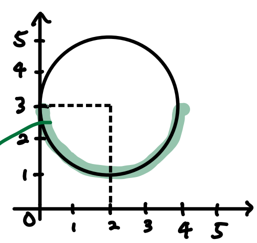
$(x^2-2x+1)+(y^2+4y+4)=8+1+4$

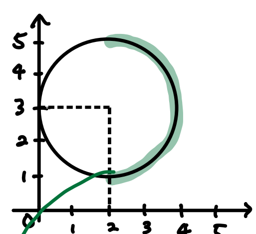
$\boxed{(x-1)^2+(y+2)^2=13=(\sqrt{13})^2}$ $\Rightarrow$ $\boxed{\text{center}: (1,-2) \quad \text{radius}: \sqrt{13}}$

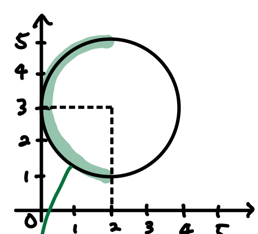
$(x-1)^2+(y-(-2))^2=(\sqrt{13})^2$

Ex Find equations for the upper half, lower half, right half, and left half of the circle $(x-2)^2+(y-3)^2=4$









All the points on the upper half have (y-coordinate) ≥ 3

$\boxed{y=3+\sqrt{-x^2+4x}}$

All the points on the lower half have (y-coordinate) ≤ 3

do the same proof

$\Rightarrow \boxed{y=3-\sqrt{-x^2+4x}}$

All the points on the right half have (x-coordinate) ≥ 2

$\boxed{x=2+\sqrt{-y^2+6y-5}}$

All the points on the left half have (x-coordinate) ≤ 2

$\boxed{x=2-\sqrt{-y^2+6y-5}}$

Solve $(x-2)^2+(y-3)^2=4$ for y

$\downarrow -(x-2)^2$

$(y-3)^2=4-(x-2)^2=4-(x^2-4x+4)$
$=-x^2+4x$

$(y-3)^2=-x^2+4x$

$y-3=\pm\sqrt{-x^2+4x}$ $\rightarrow$ $y=3+\sqrt{-x^2+4x}\ \geq 3$
or
$y=3-\sqrt{-x^2+4x}\ \leq 3$

Solve $(x-2)^2+(y-3)^2=4$ for x.

$\downarrow -(y-3)^2$

$(x-2)^2=4-(y-3)^2$
$=4-(y^2-6y+9)$
$=-y^2+6y-5$

$\downarrow$

$x-2=\pm\sqrt{-y^2+6y-5}$ $\longrightarrow$ $x=2+\sqrt{-y^2+6y-5}\ \geq 2$
or
$x=2-\sqrt{-y^2+6y-5}\ \leq 2$

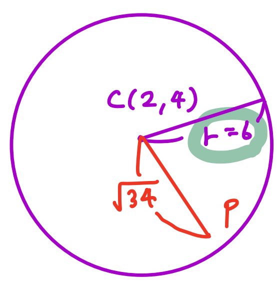
* How can one check that a point is in a circle or not?

: Compare

(the distance between the center and a point) and a radius!

Ex Determine whether the point P is inside, outside, or on the circle with center C and radius r.

$$\begin{cases} P(7,1) \\ C(2,4) \\ r=6 \end{cases}$$



$$d(C,P) = \sqrt{(7-2)^2+(1-4)^2} = \sqrt{5^2+(-3)^2}$$

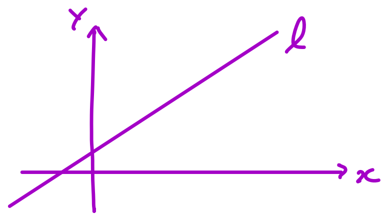
$$= \sqrt{25+9}$$

$$= \sqrt{34}$$

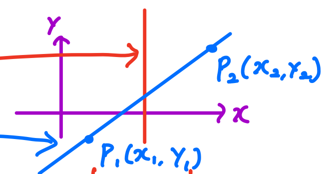
$\sqrt{34} < 6 = \sqrt{36}$

$\Rightarrow$ $\boxed{\text{P is inside!}}$

Section 3.3. Lines



Slope of a line: A quantity that represents the steepness of a line

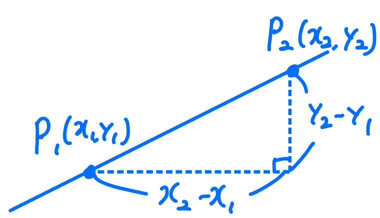


How to measure?

If a line ℓ is vertical, its slope is undefined.

If a line ℓ is not vertical and passes through two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, the slope of ℓ is defined to be

$$(\text{slope}) = \boxed{\frac{y_2 - y_1}{x_2 - x_1}}$$



Ex Find a slope of the line through

1) $A\,(\overset{x_2}{\underset{x_1}{3}}, \overset{y_2}{\underset{y_1}{5}})$ and $B\,(\overset{x_1}{\underset{x_2}{-1}}, \overset{y_1}{\underset{y_2}{11}})$ $\quad \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{11-5}{-1-3} = \dfrac{6}{-4} = \boxed{-\dfrac{3}{2}}$

$$= \frac{5-11}{3-(-1)} = \frac{-6}{4} = \boxed{-\frac{3}{2}}$$

2) $A\,(\underset{x_1}{4}, \underset{y_1}{2})$ and $B\,(\underset{x_2}{4}, \underset{y_2}{7})$ $\quad \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{7-2}{4-4} = \dfrac{5}{0}$ $\boxed{:\text{undefined}}$