Exam I : Next Wednesday (9/22), 1:45 - 2:35

Online HW 3 & Written HW 3 : due this Friday at 1 pm

Among 8 problems, 4 of them will be very similar to review exercise problems (at the end of each chapter in the textbook)

from Written HW 2

Section 2.1. #16. Solve $CD + C = PC + N$ for C.

$$-PC \qquad -PC$$

$$DC + 1C - PC = N$$

$$\downarrow$$

$$(D+1-P)\cdot C = N \xrightarrow{\div(D+1-P)} \boxed{C = \frac{N}{D+1-P}}$$

Section 2.3. #14. Solve $\frac{3x}{x-2} + \frac{1}{x+2} = \frac{-4}{x^2-4}$

STEP1 Find l.c.d. : $(x+2)\cdot(x-2)$

STEP2 When denominators become zero? $x=2$, $x=-2$.

STEP3&4 Multiply l.c.d. and solve it!

$$3x(x+2) + (x-2) = -4$$

$$\downarrow$$

$$3x^2 + 7x + 2 = 0$$

$$(3x+1)(x+2) = 0.$$

$$\to 3x+1=0 \text{ or } x+2=0$$

$$\downarrow$$

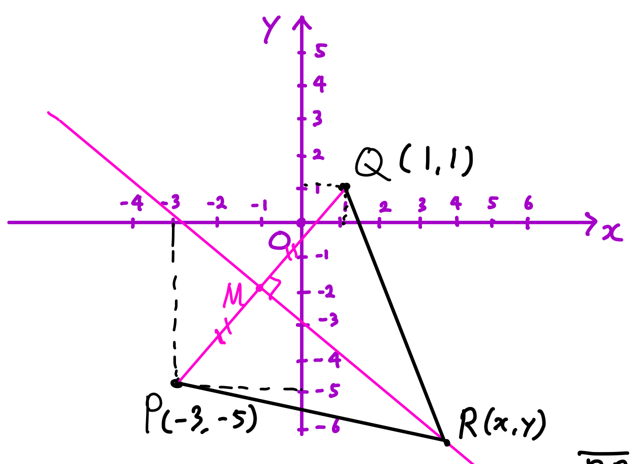
$$\boxed{x = -\frac{1}{3}} \text{ or } \cancel{x=-2}.$$

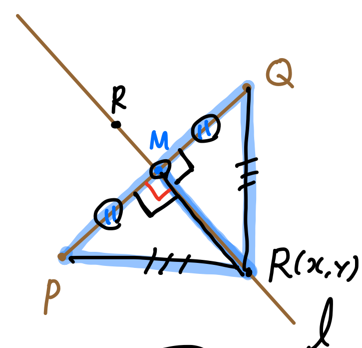
* See the "Written HW 2 Solution" to see the detail of the proof.

It is on the bottom of the course Canvas main page.

Section 3.1. Continued.

Ex Given $P(-3,-5)$ and $Q(1,1)$, find a formula that express the fact that an arbitrary point $R(x,y)$ is on the perpendicular bisector ℓ of segment PQ.





△PRM and △QRM are equal.

$\Rightarrow \boxed{\overline{PR} = \overline{QR}}$

$$\overline{PR} = \sqrt{(x-(-3))^2+(y-(-5))^2}$$

$$= \sqrt{(x+3)^2+(y+5)^2}$$

$$\overline{QR} = \sqrt{(x-1)^2+(y-1)^2}$$

$$\sqrt{(x+3)^2+(y+5)^2} = \sqrt{(x-1)^2+(y-1)^2}$$

$\downarrow$ square.

$$(x+3)^2+(y+5)^2 = (x-1)^2+(y-1)^2$$

$$\cancel{x^2}+6x+9+\cancel{y^2}+10y+25 = \cancel{x^2}-2x+1+\cancel{y^2}-2y+1$$

$$8x+12y+32=0.$$

$\downarrow \div 4$

$$\boxed{2x+3y+8=0.}$$

Section 3.2. Graphs of Equations (in two variables x and y)

- Solution of equation in x and y

: An ordered pair (a, b) that yield a true statement if $x=a$, $y=b$.

Ex $x-2y+1=0 \Rightarrow (x,y)=(1,1), (3,2), \cdots$: there are infinitely many solutions!

$1-2\cdot1+1=1-2+1=0$

- Graph of equation in x and y

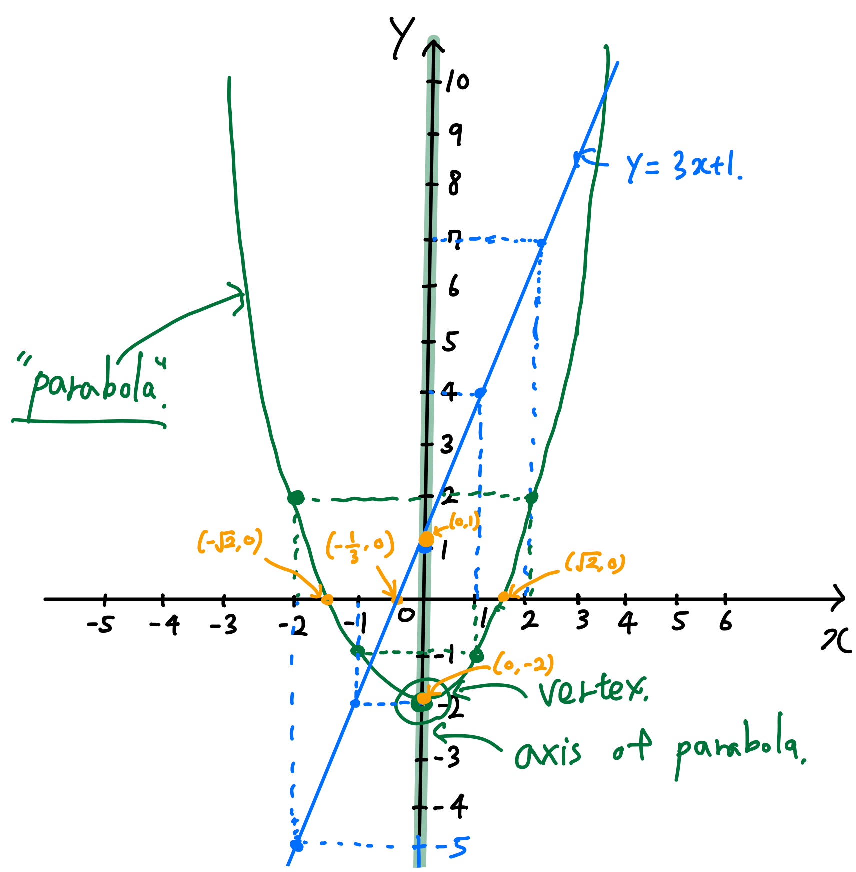
: All solutions of the equation on the coordinate plane.

$y=3x+1$

x	y
(-2,	-5)
(-1,	-2)
(0,	1)
(1,	4)
(2,	7)

$y=x^2-2$

x	y
(-2,	2)
(-1,	-1)
(0,	-2)
(1,	-1)
(2,	2)



$(?, \underline{0})$ $(\underline{0}, ?)$

x-intercept and y-intercept of the graph.

intersection of the graph and x-axis
Replace y by 0, and then solve for x.

intersection of the graph and y-axis
Replace x by 0, and then solve for y.

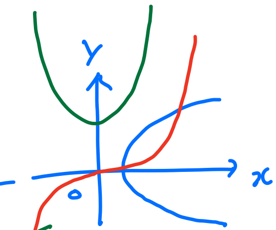
Ex Find x- and y-intercept.

① $y = 3x+1$: x-intercept : $0 = 3x+1 \to$, $x = -\frac{1}{3} \Rightarrow (-\frac{1}{3}, 0)$
y-intercept : $y = 0+1 = 1 \longrightarrow (0, 1)$

② $y = x^2 - 2$: x-intercept : $0 = x^2 - 2$, $x^2 = 2$, $x = \pm\sqrt{2} \to (\sqrt{2}, 0), (-\sqrt{2}, 0)$
y-intercept : $y = 0 - 2 = -2 \longrightarrow (0, -2)$

see also the graphs on the previous page :
We plot these x-intercepts and y-intercepts.

Symmetry of Graphs



Graph of an equation is

① Symmetric w.r.t. x-axis if the equation remains the same when $y \to -y$

They are mirror symmetry

② Symmetric w.r.t. y-axis if the equation remains the same when $x \to -x$

It is a rotational symmetry

③ Symmetric w.r.t. the origin if the equation remains the same when $\begin{cases} x \to -x \\ y \to -y \end{cases}$

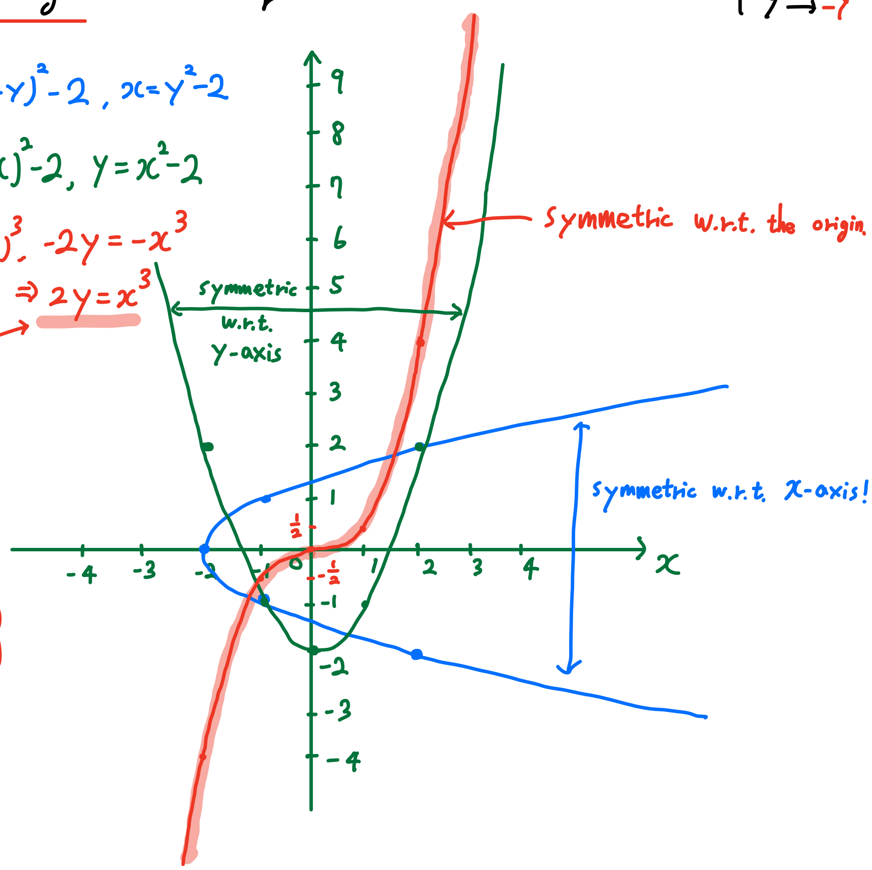
$x = y^2 - 2$: $x = (-y)^2 - 2$, $x = y^2 - 2$

$y = x^2 - 2$: $y = (-x)^2 - 2$, $y = x^2 - 2$

$2y = x^3$: $2(-y) = (-x)^3$, $-2y = -x^3$

$\Rightarrow 2y = x^3$

the same!

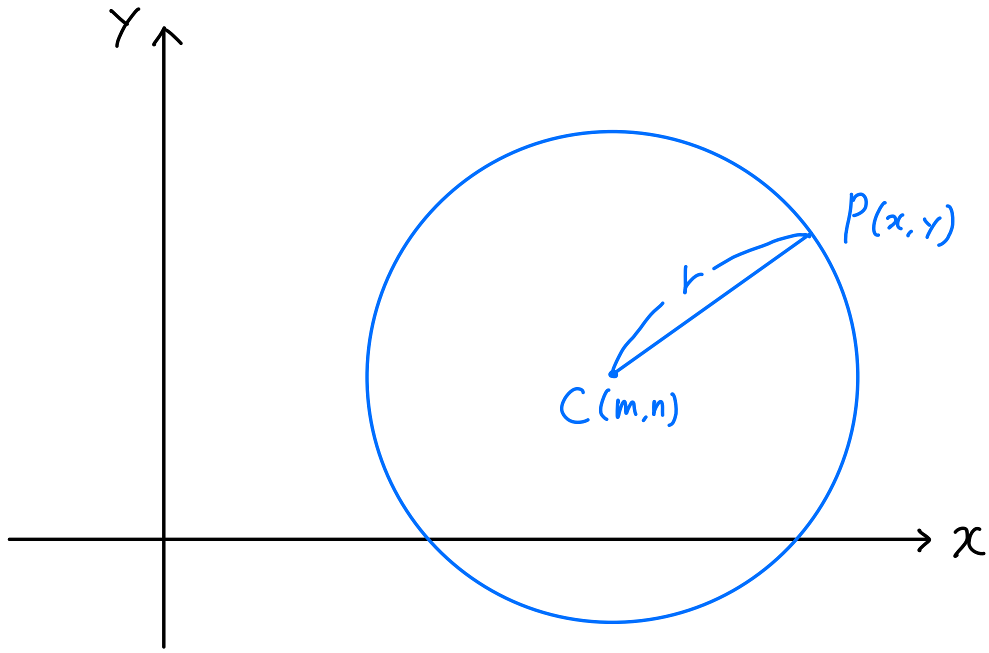


x	y
(2	, -2)
(-1	, -1)
(-2	, 0)
(-1	, 1)
(2	, 2)

x	y
(-2,	2)
(-1,	-1)
(0,	-2)
(1,	-1)
(2,	2)

x	y
(-2,	-4)
(-1,	$-\frac{1}{2}$)
(0,	0)
(1,	$\frac{1}{2}$)
(2,	4)

Equation of a circle



By the distance formula: $\sqrt{(x-m)^2+(y-n)^2} = r$

$\downarrow$

$$\boxed{(x-m)^2+(y-n)^2 = r^2}$$: Standard equation of a circle

standard equation of a circle with (center) = (m, n) and (radius) = r