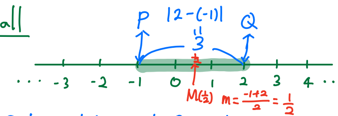
Chapter 3. Functions and Graphs

Section 3.1. Rectangular Coordinate System.

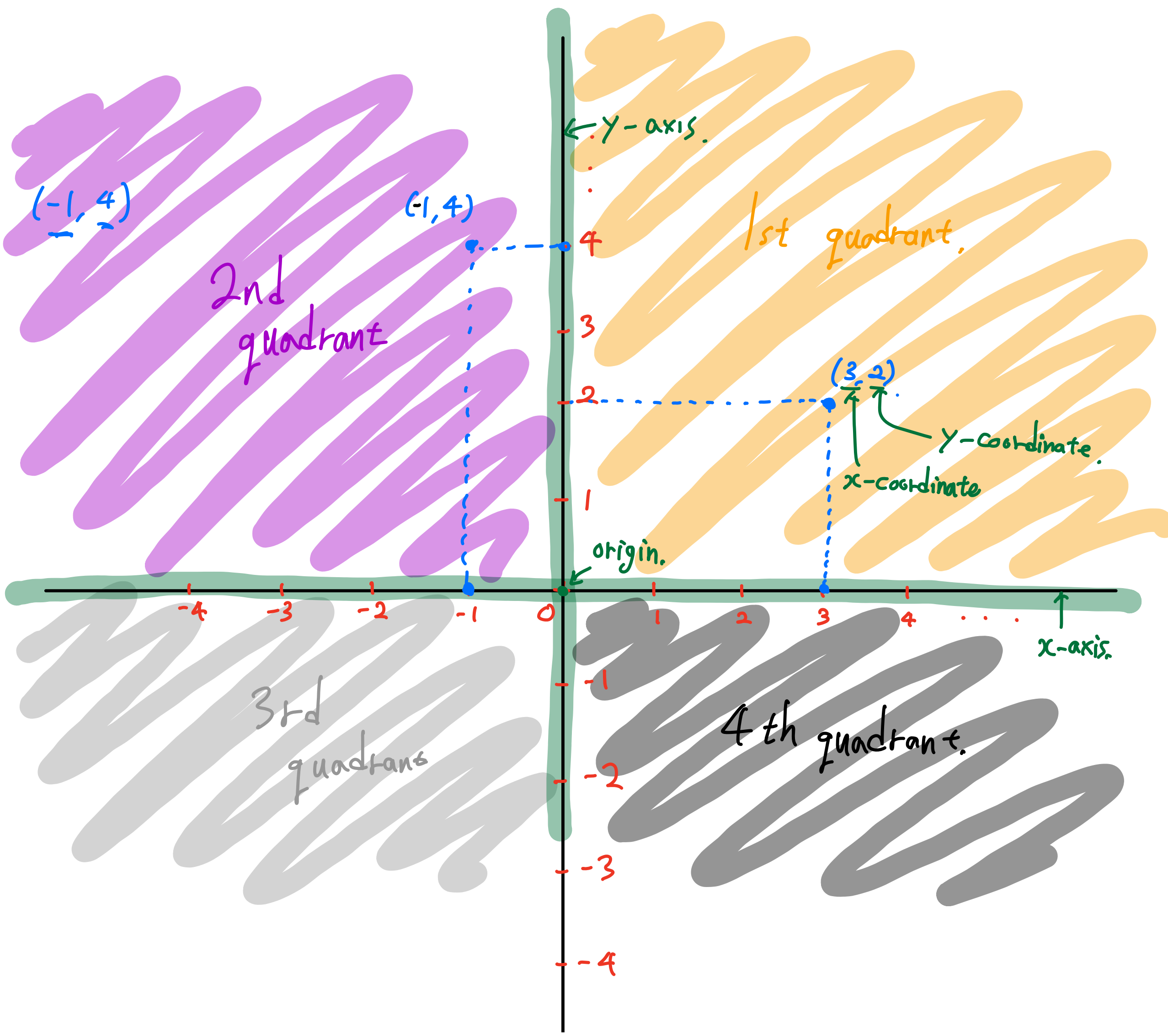
Recall



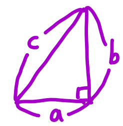
① Let $P(x)$ and $Q(y)$ be two points on the number line.
Then the distance between two points is $d(P, Q) = |y - x|$.

② Let $M(m)$ be the midpoint of the segment PQ. Then $m = \frac{x+y}{2}$

Rectangular Coordinate System : 2D version of a number line.



Two important formulas :



$\Rightarrow c^2 = a^2 + b^2$

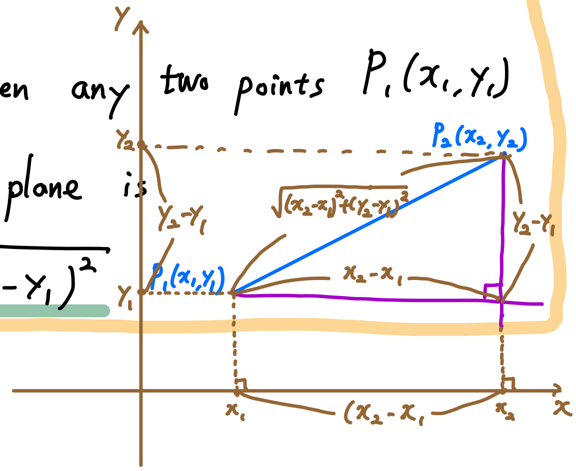
$c = \sqrt{a^2+b^2}$

Pythagorean Theorem!

1) Distance Formula

: The distance $d(P_1, P_2)$ between any two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in a coordinate plane is

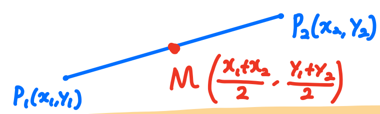
$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



2) Midpoint Formula

: The midpoint M of the line segment from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

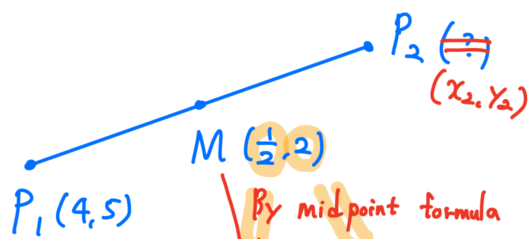


We will see several examples that can be solved by using the above formulas!

Ex If $M(\frac{1}{2}, 2)$ is the midpoint of the line segment from $P_1(4, 5)$ to P_2.

① what is the coordinate of P_2? Let $P_2(\underset{\downarrow}{\overset{-3}{x_2}}, \underset{\downarrow}{\overset{-1}{y_2}}) \Rightarrow \boxed{(-3, -1)}$

② what is $d(M, P_2)$?



By midpoint formula

$$\left(\frac{4+x_2}{2}, \frac{5+y_2}{2}\right)$$

① $\frac{4+x_2}{2} = \frac{1}{2}$, $\frac{5+y_2}{2} = 2$.

$\downarrow \times 2$ $\qquad$ $\downarrow \times 2$

$4 + x_2 = 1$ $\qquad$ $5 + y_2 = 4$

$\downarrow -4$ $\qquad$ $\downarrow -5$

$x_2 = -3$ $\qquad$ $y_2 = -1$

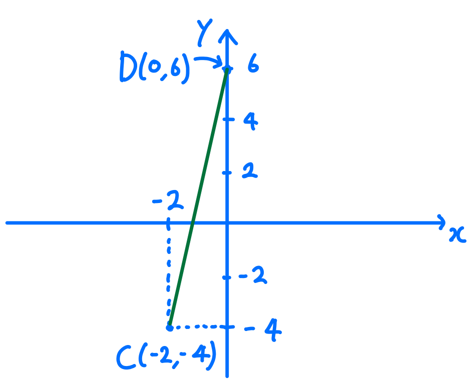
② $M(\underset{x_1}{\underline{\tfrac{1}{2}}}, \underset{y_1}{\underline{2}})$, $P_2(\underset{x_2}{\underline{-3}}, \underset{y_2}{\underline{-1}})$ $\xrightarrow[\text{formula}]{\text{distance}}$ $\sqrt{(-3-\frac{1}{2})^2 + (-1-2)^2}$

$= \sqrt{(-\frac{7}{2})^2 + (-3)^2}$

$= \sqrt{\frac{49}{4} + 9} = \sqrt{\frac{49}{4} + \frac{36}{4}} = \sqrt{\frac{85}{4}}$ $= \boxed{\frac{\sqrt{85}}{2}}$

DIY

Ex Plot the points $C(-2, -4)$ and $D(0, 6)$, and find the distance $d(C, D)$



By the distance formula,

$d(C, D) = \sqrt{(0-(-2))^2 + (6-(-4))^2}$

$= \sqrt{2^2 + 10^2}$

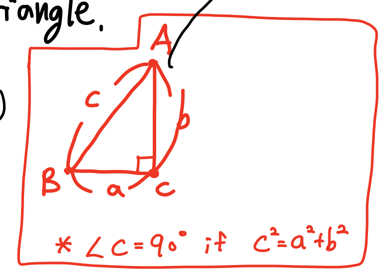
$= \sqrt{4 + 100}$

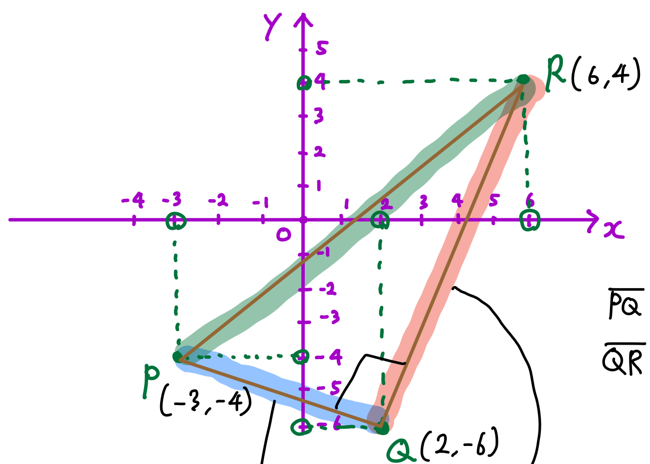
$= \sqrt{104}$ $= \sqrt{4 \cdot 26}$

$= \boxed{2\sqrt{26}}$

Ex Plot $P(-3,-4)$, $Q(2,-6)$, and $R(6,4)$ and show that $\triangle PQR$ is a right triangle. Then find the area of the triangle.

area $= \frac{1}{2}\cdot a\cdot b$.





$\overline{PQ} = \sqrt{(2-(-3))^2+(-6-(-4))^2} = \sqrt{5^2+(-2)^2} = \sqrt{29}$

$\overline{QR} = \sqrt{(6-2)^2+(4-(-6))^2} = \sqrt{4^2+10^2} = \sqrt{116}$
$= \sqrt{4\cdot 29}$
$= 2\cdot\sqrt{29}$

$\overline{PR} = \sqrt{(6-(-3))^2+(4-(-4))^2} = \sqrt{9^2+8^2} = \sqrt{81+64}$
$= \sqrt{145}$

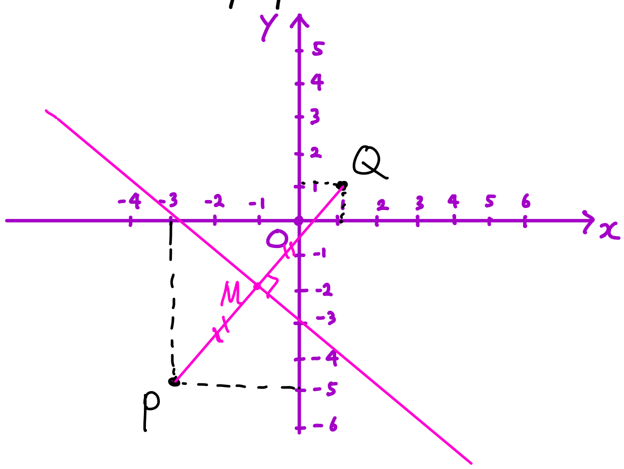
$(\text{area}) = \frac{1}{\cancel{2}_1}\cdot\sqrt{29}\cdot\cancel{2}^1\cdot\sqrt{29}$
$= \boxed{29}$ //

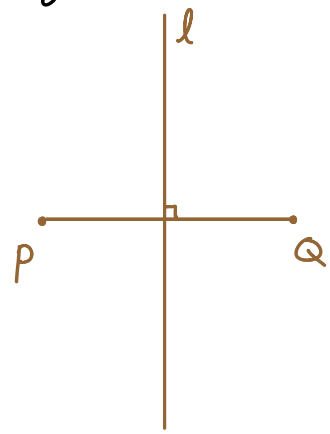
$\Rightarrow$ Claim $\overline{PQ}^2 + \overline{QR}^2 = (\sqrt{29})^2 + (\sqrt{116})^2$
$= 29 + 116$
$= 145$
$= (\sqrt{145})^2$
$= \overline{PR}^2$

$\Rightarrow \angle PQR$ is a right angle

$\Rightarrow \triangle PQR$ is a right triangle.

<u>Ex</u> Given $P(-3,-5)$ and $Q(1,1)$, find a formula that express the fact that an arbitrary point $R(x,y)$ is on the perpendicular bisector ℓ of segment PQ.





: We stopped here!