# Section 2.7. More on Inequalities.

HW 3 will be posted (Today 5pm).

Solving { quadratic inequalities
rational inequalities.

Ex $\dfrac{\overset{\text{Negative,}}{\boxed{-2}}}{3-x} \geq 0 \Rightarrow 3-x$ is negative

$\Rightarrow 3-x<0$

$\Rightarrow \boxed{3<x}$

## Solving quadratic inequalities

(Later, we will see another proof: using the graph of the quadratic function)

**Step 1** Rewrite the inequality in one of the following form : $\left\{\begin{array}{l} ax^2+bx+c>0 \\ \boxed{ax^2+bx+c\geq 0} \\ ax^2+bx+c<0 \\ ax^2+bx+c\leq 0 \end{array}\right.$
and divide the inequality by the g.c.f of a, b, and c. (or − g.c.f.)

**Step 2** Factor the left hand side : $\left\{\begin{array}{l} (px+q)(rx+s)>0 \\ (px+q)(rx+s)\geq 0 \\ (px+q)(rx+s)<0 \\ (px+q)(rx+s)\leq 0 \end{array}\right.$

**Step 3** Find the values of $x$ that makes (one of the factor) = 0 : $\begin{array}{l} px+q=0 \\ rx+s=0 \end{array}$

**Step 4** Partition the real line using the values of $x$ from Step 3.

**Step 5** Choose the test value $k$ from each part, and check whether the inequality holds or not. (or observe the sign change of each factor and decide the sign of the given expression)

**Step 6** Takes the union of all parts whose test value makes inequality true.

**Step 7** Add values of $x$ from Step 3 if original inequality has $\geq$ or $\leq$.

## Ex Solve $-6x^2 \geq -4x-10$

**STEP1** $+4x+10 \qquad +4x+10$

$-6x^2+4x+10 \geq 0$

$\div(-2)$

$3x^2-2x-5 \leq 0$.

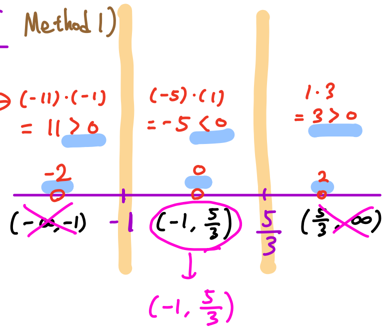
**STEP2** Factor it! $(3x-5)(x+1) \leq 0$

**STEP3** $3x-5=0 \longrightarrow \boxed{x=\frac{5}{3}}$

$x+1=0 \longrightarrow \boxed{x=-1}$

**STEP4**

**STEP5** Method 1)



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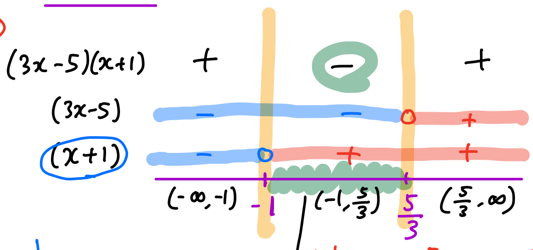
**STEP5** Method 2).



when $x<-1$, $x+1<0$

$x>-1$, $x+1>0$

When $x>\frac{5}{3}$, $3x-5>0$

When $x<\frac{5}{3}$ $3x-5<0$

$(-1,\frac{5}{3})$.

**STEP6** Take the union of $(-1,\frac{5}{3})$

$\Rightarrow (-1,\frac{5}{3})$

**STEP7**. add $-1$ and $\frac{5}{3}$ to $(-1,\frac{5}{3})$

$\Rightarrow \boxed{[-1,\frac{5}{3}],\ -1 \leq x \leq \frac{5}{3}}$

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DIY.

## Ex Solve $-3x^2-15 < 18x$

$-18x$

**STEP1**. $-3x^2-18x-15 < 0$

$\div 3$

$x^2+6x+5 > 0$

factor!

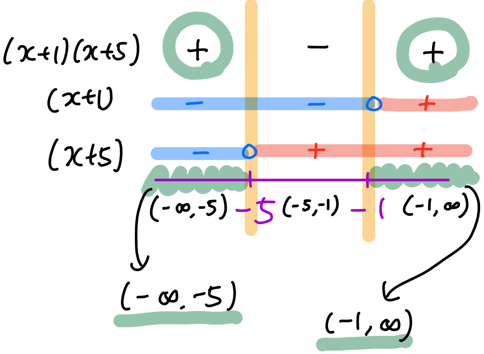
**STEP2**. $(x+1)(x+5) > 0$

**STEP3** $x+1=0$ when $\boxed{x=-1}$

$x+5=0$ when $\boxed{x=-5}$

**STEP4**

**STEP5**



**STEP 6** Take the union!

$\boxed{(-\infty,-5)\cup(-1,\infty)}$, $\boxed{x>-1 \text{ or } x<-5}$

We omit STEP 7 because inequality symbol is not $\geq, \leq$.

# Solving inequalities involving rational expression.

* Almost the same approach works!

* Make sure to factor each part as much as you can!

Ex Solve $\frac{(x-4)(x+1)}{(x-1)(x^2-4)} \geq 0$

(Skip STEP1 and STEP2) ↓ factor $x^2-4$

$$\frac{(x-4)(x+1)}{(x-1)(x+2)(x-2)} \geq 0$$

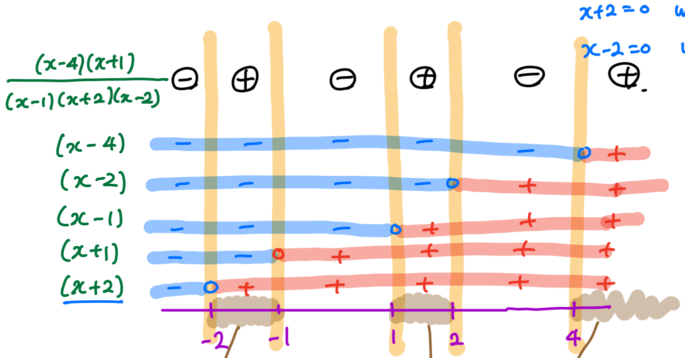
$x-4=0$ when $x=4$ ✓

$x+1=0$ when $x=-1$ ✓

$x-1=0$ when $x=1$ ×

$x+2=0$ when $x=-2$ ×

$x-2=0$ when $x=2$ × } because they make (denominator) $=0$.



when $x<-2$, $x+2<0$

$x>-2$, $x+2>0$

$(-2,-1)\cup(1,2)\cup(4,\infty)$

↓ add −1 and 4

$\boxed{[-2,-1]\cup(1,2)\cup[4,\infty)}$

DIY!

Ex Solve $\frac{(3x-2)^2(x+2)}{x(x^2+4)} < 0$

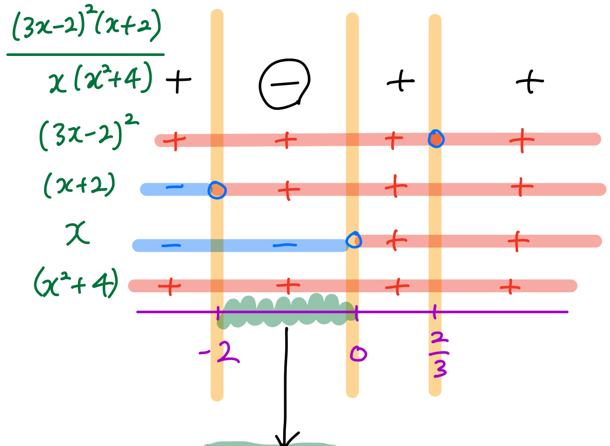
We cannot factor further!

$3x-2=0$ when $\boxed{x=\frac{2}{3}}$

$x+2=0$ when $\boxed{x=-2}$

$x=0$ when $\boxed{x=0}$

$x^2+4$ cannot be zero because $x^2+4 \geq 0+4=4$.



: $(3x-2)^2$ is always nonnegative because it is a square of $(3x-2)$!

$\boxed{(-2,0)}$ : We do not care about $x=\frac{2}{3}$, $x=-2$, $x=0$ because inequality symbol is not $\geq, \leq$.

Ex Solve $\frac{x}{3x+2} \ge \frac{1}{x+2}$

$$\Big\downarrow \; -\frac{1}{x+2}$$

$$\frac{x}{3x+2} - \frac{1}{x+2} \ge 0$$

$$\Big\downarrow$$

$$\frac{x\cdot(x+2)}{(3x+2)(x+2)} - \frac{1\cdot(3x+2)}{(3x+2)(x+2)} \ge 0.$$

$$\frac{x\cdot(x+2) - 1\cdot(3x+2)}{(3x+2)(x+2)} \ge 0. \longrightarrow \frac{x^2+2x-3x-2}{(3x+2)(x+2)} \ge 0$$

$$\frac{x^2-x-2}{(3x+2)(x+2)} \ge 0.$$

$$\frac{(x-2)(x+1)}{(3x+2)(x+2)} \ge 0.$$

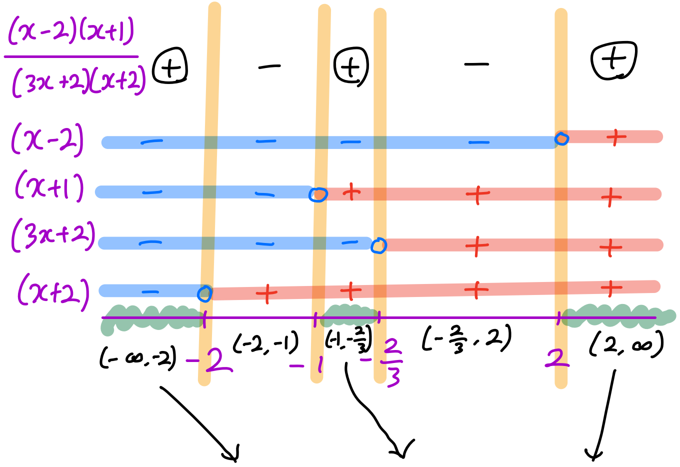
$x-2=0$ when $\boxed{x=2}$

$x+1=0$ when $\boxed{x=-1}$

$3x+2=0$ when $\cancel{x=-\frac{2}{3}}$

$x+2=0$ when $\cancel{x=-2}.$

↳ They make (denominator) $=0$.



$$(-\infty,-2) \cup (-1,-\tfrac{2}{3}) \cup (2,\infty)$$

$\Big\downarrow$ add $x=2$ and $x=-1$

$$\boxed{(-\infty,-2) \cup [-1,-\tfrac{2}{3}) \cup [2,\infty)}$$