

Do not forget two Homework (due tomorrow at 1pm)

## Section 2.6. Inequalities

$$\text{Solving equation: } 3x - 7 = 2x - 5 \longrightarrow x = 2$$

$$\text{Solving inequality: } 3x - 7 \geq 2x - 5 \longrightarrow x \geq 2$$

Inequality: a statement that two quantities or expressions are not equal.

$a > b$ :  $a$  is greater than  $b$ .  $a \geq b$ ,  $a < b$ ,  $a \leq b$ .

We want to "solve" inequalities using the following properties.

Adding / Subtracting the same real number preserves the inequality.

Multiplying / Dividing the same positive real number preserves the inequality.

Multiplying / Dividing the same negative real number flips the inequality.

$$-2x > 6 \xrightarrow{\div (-2)} \frac{-2x}{-2} < \frac{6}{-2}$$

You should be able to express your answer in

① interval notation, ② inequality, or ③ graph. (see p. 103 of the textbook)

$$[-2, \infty)$$

$$x \geq -2$$



$a < b < c$  means " $a < b$  and  $b < c$ "

$$\underline{\text{Ex}} \quad -3x + 7 > 2x - 8 \quad : \quad \underline{-3x + 7} = \underline{2x - 8}$$

$$\begin{array}{r} -2x \qquad -2x \\ -5x + 7 > -8 \\ -7 \qquad -7 \\ -5x > -15 \\ \downarrow \div (-5) \\ \frac{-5x}{(-5)} < \frac{-15}{-5} \Rightarrow \boxed{x < 3} \end{array}$$

$$\begin{array}{r} -5x + 7 = -8 \\ -7 \qquad -7 \\ -5x = -15 \\ \downarrow \div (-5) \\ \boxed{x = 3} \end{array}$$

$$\underline{\text{Ex}} \quad -7 < \frac{5-4x}{3} \leq 1 \quad \longrightarrow \quad -7 < \frac{5-4x}{3} \text{ and } \frac{5-4x}{3} \leq 1$$

Proof 1

$$\begin{array}{l} (-7) \times 3 < 3 \times \left(\frac{5-4x}{3}\right) \leq 1 \times 3 \\ -21 < 5-4x \leq 3 \\ \downarrow -5 \\ -26 < -4x \leq -2 \\ \downarrow \div (-4) \\ \frac{(-26)}{(-4)} > \frac{(-4x)}{(-4)} \geq \frac{(-2)}{(-4)} \\ \boxed{\frac{13}{2} > x \geq \frac{1}{2}} \end{array}$$

Proof 2

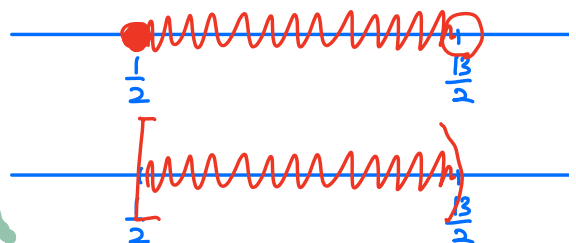
$$\begin{array}{l} (-7) \times 3 < 3 \times \left(\frac{5-4x}{3}\right) \\ \downarrow \\ -21 < 5-4x \\ \downarrow -5 \\ -26 < -4x \\ \downarrow \div (-4) \\ \frac{(-26)}{(-4)} > \frac{(-4x)}{(-4)} \\ \boxed{\frac{13}{2} > x \text{ and } x \geq \frac{1}{2}} \\ \downarrow \\ \frac{1}{2} \leq x \text{ and } x < \frac{13}{2} \\ \downarrow \\ \boxed{\frac{1}{2} \leq x < \frac{13}{2}} \end{array}$$

$$\underline{\text{Ex}} \quad \frac{-2}{3-x} \geq 0$$

← Negative.

Negative  $\geq 0 \Rightarrow$  denominator is ~~negative~~ positive.

$$\begin{array}{l} \Rightarrow 3-x \leq 0 \\ +x \quad +x \\ 3 \leq x \rightarrow \boxed{3 < x} \end{array}$$



Union  $\leftrightarrow$  "or"  
 intersection  $\leftrightarrow$  "and"



Union ( $\cup$ ): "All"

Ex)  $(1, 3]$  or  $(2, 4) \Rightarrow (1, 3] \cup (2, 4) = (1, 4)$

Intersection ( $\cap$ ): "Common"

Ex)  $(1, 3]$  and  $(2, 4) \Rightarrow (1, 3] \cap (2, 4) = (2, 3]$



For any positive real number  $b$ ,

$|a| < b$ :  $\Rightarrow -b < a < b$

A number line with points  $-b$ ,  $a$ ,  $0$ , and  $b$  marked. The interval between  $-b$  and  $b$  is shaded green. A blue arrow points to  $a$  with the text "should lie between  $-b$  and  $b$ ".

$|a| > b$ :  $\Rightarrow a > b$  or  $a < -b$

A number line with points  $-b$ ,  $0$ , and  $b$  marked. The intervals  $(-\infty, -b)$  and  $(b, \infty)$  are shaded green.

Ex ①  $|2x - 1| < \frac{5}{3}$

$\Rightarrow -\frac{5}{3} < 2x - 1 < \frac{5}{3}$

$\downarrow +1 = +\frac{3}{3}$

$-\frac{2}{3} < 2x < \frac{8}{3}$

$\downarrow \times \frac{1}{2}$

$(-\frac{2}{3}) \cdot \frac{1}{2} < \frac{1}{2} \cdot 2x < \frac{8}{3} \cdot \frac{1}{2}$

$-\frac{1}{3} < x < \frac{4}{3}$

②  $|2x - 1| < -\frac{5}{3}$

Non-negative. ~~X~~ Negative

: No solution.

②'  $|2x - 1| > -\frac{5}{3}$

: All real numbers.

$$\underline{\text{Ex}} \text{ ③ } 3|2-x|-2 > 7$$

$$\begin{array}{c} +2 \quad +2 \\ 3|2-x| > 9 \end{array}$$

$$\downarrow \div 3$$

$$|2-x| > 3$$

$$\downarrow$$

$$2-x > 3 \quad \text{or} \quad 2-x < -3$$

$$\downarrow$$

$$\boxed{x < -1 \quad \text{or} \quad 5 < x}$$

~~$$5 < x < 2$$~~

~~$$5 < x < 2$$~~

$$\text{④ } \frac{5}{|3-2x|} > 1$$

$3-2x=0 \rightarrow 2x=3 \quad x=\frac{3}{2}$   
 1) Denominator  $\neq 0 \Rightarrow 3-2x \neq 0 \Rightarrow x \neq \frac{3}{2}$   
 2) If  $3-2x \neq 0, |3-2x| > 0$

$$\frac{5}{|3-2x|} > 1$$

$$\downarrow \times |3-2x|, \text{ which is positive!}$$

$$5 > |3-2x| \rightarrow |3-2x| < 5$$

$$-5 < 3-2x < 5$$

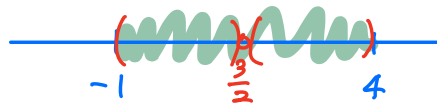
$$\downarrow -3$$

$$-8 < -2x < 2$$

$$\downarrow \div 2$$

$$\boxed{-1 < x < 4} \quad \text{and} \quad x \neq \frac{3}{2}$$

it is not the answer because it includes  $x = \frac{3}{2}$



$$\boxed{-1 < x < \frac{3}{2} \quad \text{or} \quad \frac{3}{2} < x < 4}$$

\*  $a < c < b$  makes sense only if  $a < b$

If  $5 < x$  and  $x < 2$ , there is no such  $x$  so No solution!

\* Never use  $a < c > b$  or  $a > c < b$

If  $2 < x$  and  $x > 5$ , we can just say  $x > 5$ .

If  $2 > x$  and  $x < 5$ , we can just say  $2 > x$