We will skip Section 2.4 (comeback on Dec)

If you feel sick, join the Zoom: 830 280 9924 password : sw007

Section 2.5. Other Types of Equations

$|x| = a \Rightarrow x = a$ or $x = -a$

* Equations involving absolute value symbols, rational exponents, ...

Ex Solve : $3|x-2| - 2 = 7$

$+2 \quad +2$

$3|x-2| = 9$

$\downarrow \div 3$

$|x-2| = 3$

$\downarrow$

$x-2 = 3$ or $x-2 = -3 \longrightarrow$ $x = 5$ or $x = -1$

Ex Solve : $2|3x+1| - 3 = 5$

$+3 \quad +3$

$2|3x+1| = 8$

$\downarrow \div 2$

$|3x+1| = 4$

$\downarrow$

$3x+1 = 4$ or $3x+1 = -4$

$\downarrow$

$3x = 3$ or $3x = -5$

$\downarrow$

$x = 1$ or $x = -\frac{5}{3}$

* $|x| = -1$ ↑ No solution!

Ex Solve: $2x^3 - 3x^2 - 2x + 3 = 0$

$\Rightarrow (2x^3 - 2x) + (-3x^2 + 3) = 0$

$\Rightarrow 2x(x^2 - 1) - 3(x^2 - 1) = 0$

$\Rightarrow (x^2 - 1)(2x - 3) = 0 \xrightarrow{\text{Z.F.T.}} x^2 - 1 = 0$ or $2x - 3 = 0$

$\Rightarrow (x+1)(x-1)(2x-3) = 0$

$(x+1)(x-1) = 0$

Z.F.T.

$x + 1 = 0$ or $x - 1 = 0$

$x = -1$ $\quad x = 1$

$x = \frac{3}{2}$

Ex Solve: $x^{\frac{3}{2}} - 4x^{\frac{1}{2}} = 0$

$= x \cdot x^{\frac{1}{2}} - 4x^{\frac{1}{2}} = x^{\frac{1}{2}}(x - 4) = 0$

Z.F.T.

$x^{\frac{1}{2}} = 0$ or $x - 4 = 0 \longrightarrow \boxed{x = 0 \text{ or } x = 4}$

Ex Solve: $3 = x - \sqrt{5 - x}$: one radical in the equation.

$-x \quad -x$ isolate it!

$3 - x = -\sqrt{5 - x}$

taking squares.

$(3 - x)^2 = \left(-\sqrt{5 - x}\right)^2$

$3^2 - 2 \cdot 3 \cdot x + x^2 = 5 - x$

$9 - 6x + x^2 = 5 - x$

$-(5 - x) \quad -(5 - x)$

$-5 + x$

$x^2 - 5x + 4 = 0$

$(x - 1)(x - 4) = 0 \xrightarrow{\text{Z.F.T.}} x - 1 = 0$ or $x - 4 = 0 \longrightarrow x = 1$ or $x = 4$

If you take square here,

$3^2 = \left(x - \sqrt{5 - x}\right)^2$

$9 = x^2 - 2 \cdot x \cdot \sqrt{5 - x} + (5 - x)$

Check: $3 = 1 - \sqrt{5 - 1}$

$3 = 1 - 2$: NOT TRUE!

$\Rightarrow x = 1$ is NOT a solution!

Check: $3 = 4 - \sqrt{5 - 4}$

$3 = 4 - 1$: TRUE!

$x = 4$ is a solution!

$x=0$: $\sqrt{1}-\sqrt{4}=1$: False, $x=0$ is NOT a solution.

$x=5$: $\sqrt{3\cdot5+1}-\sqrt{5+4}=1$; $\sqrt{16}-\sqrt{9}=1$; TRUE $x=5$ is a solution!

Ex Solve : $\sqrt{3x+1}-\sqrt{x+4}=1$ two radicals in the equation.

(split them!)

$+\sqrt{x+4}$ $+\sqrt{x+4}$

$\sqrt{3x+1}=1+\sqrt{x+4}$

↓ square.

$(a+b)^2=a^2+2ab+b^2$

$(1+\sqrt{x+4})^2=1^2+2\cdot1\cdot\sqrt{x+4}+x+4$

$3x+1=1+2\cdot1\cdot\sqrt{x+4}+x+4$

$3x+1=2\sqrt{x+4}+x+5$

$-x-5$ $-x-5$

$2x-4=2\sqrt{x+4}$

↓ ÷2

$x-2=\sqrt{x+4}$ → $(x-2)^2=x+4$

$x^2-4x+4=x+4$

$-x-4$ $-x-4$

$x^2-5x=0$

$x(x-5)=0$

Z.F.T.

$x=0$ or $x=5$

Ex ① Solve : $x^{\frac{2}{3}}-4x^{\frac{1}{3}}-5=0$

$(x^{\frac{1}{3}})^2-4\cdot x^{\frac{1}{3}}-5=0$.

Let $a=x^{\frac{1}{3}}$: $a^2-4a-5=0$ → $(a-5)(a+1)=0$

→ $a-5=0$ or $a+1=0$

→ $a=5$ or $a=-1$

→ $x^{\frac{1}{3}}=5$ or $x^{\frac{1}{3}}=-1$

→ $x=5^3$ or $x=(-1)^3$

→ $x=125$ or $x=-1$

② Solve : $x^4 + 5x^2 + 3 = 0$

$(x^2)^2 + 5 \cdot x^2 + 3 = 0$

Let $b = x^2$: $\underset{a}{1} \cdot b^2 \underset{b}{+5} b \underset{c}{+3} = 0$: Cannot factor... Quadratic Formula!

$$\rightarrow b = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$

$$\rightarrow b = \frac{-5 \pm \sqrt{25 - 12}}{2}$$

$$\rightarrow b = \frac{-5 \pm \sqrt{13}}{2}$$

$$\rightarrow x^2 = \frac{-5 \pm \sqrt{13}}{2}$$

$$\rightarrow x^2 = \frac{-5 + \sqrt{13}}{2} \text{ or } x^2 = \frac{-5 - \sqrt{13}}{2}$$

Negative numbers!

x^2 cannot be a negative number...

⇒ No Real Solution!