* You cannot postpone doing your two homework!!!

Section 2.2. Continued

Recall that we need to follow the steps below:

Step 1. You should set x = (unknown quantity that the problem is asking)

Step 2. Draw a picture if it is helpful.

Step 3. Using the given condition, construct an equation.

Step 4. Solve the equation.

Step 5. Check whether the solution makes sense or not.

Today, we will see many examples of applied problems!

Ex (Calculating a presale price)

* (discount) = (presale price) × (discount rate).
  (presale price) − (discount) = (sale price)

A bookstore holding a clearance sale that all prices have been discounted 15%. If a book is on sale for $34, what was its presale price?

↳ (discount rate) = 15%.

↳ (sale price) = $34.

Step 1 Let x = the presale price.

Step 2 skip.

Step 3 ① (discount) = (presale price) × (discount rate) → (discount) = $x \cdot \frac{15}{100}$

($0.15 = \frac{15}{100}$, 15%.)

Step 4 ② (presale price) − (discount) = (sale price)

x , $x \cdot \frac{15}{100}$, 34

$$\Rightarrow x - \frac{15}{100}x = 34$$

$$1 \cdot x - \frac{3}{20}x = 34$$

$$\left(\frac{20}{20} - \frac{3}{20}\right)x = 34$$

$$\frac{\cancel{20}}{\cancel{17}} \times \frac{\cancel{17}}{\cancel{20}}x = \cancel{34}^{2} \times \frac{20}{\cancel{17}_{1}}$$

$$x = 40$$

$$\Rightarrow \text{(presale price)} = \$40.$$

Ex (Investing money in two stocks)

* **Simple Interest Formula**

If a sum of money $P (the principal) is invested at a simple interest rate r (expressed as a decimal), then the simple intest I at the end of t years is

$$I = Prt.$$

Not a percent.

rate is 10%.

It changes P every year!

Simple Interest vs Compound Interest.

$100

After 1 year... I get $10. ← $100 × 0.1 × 1

After 2 year... I get $20.

Now	1 year later	2 years later	
$100.	$110.	$120.	: Simple interest.
$100.	$110.	$121.	: Compound interest.

$I = Prt$
$= \$110 \cdot 0.1 \cdot 1 = \$11.$

$I = \$121 \cdot 0.1 \cdot 1$
$= \$12.1$

An investment firm has $100,000 to invest for a client (Total amount of money that it invest) and decides to invest in two stocks, A and B. The expected annual rate of return, or simple interest, for stock A is 20% but there is some risk involved, and the client does not wish to invest more (to stock A) than $50,000 in this stock. The annual rate of return on the more stable stock B is anticipated to be 10%. Determine whether there is a way of investing the money so that the annual interest is (a) $17,000 NO! (b) $14,000 Yes!

Step1. Let $x be the amount of money that the firm is investing to the stock A.

Claim: $(100,000 - x) is the amount of money that the firm is investing to the stock B.

Step2.

	rate	amount of money	annual interest
Stock A	20%	$x	$x \cdot \frac{20}{100} \cdot 1$
Stock B	10%	$(100,000 - x)$	$(100,000-x) \cdot \frac{10}{100} \cdot 1$

$I = P \cdot r \cdot t$ (t = 1)

(Total Interest) $= x \cdot \frac{20}{100} \cdot 1 + (100,000-x) \cdot \frac{10}{100} \cdot 1$

Step 3. (a) $(\text{Total Interest}) = x \cdot \frac{20}{100} \cdot 1 + (100{,}000 - x) \cdot \frac{10}{100} \cdot 1 = 17{,}000$.

& Step 4. (multiply 100 on both-hand side!) $\Rightarrow x \cdot \frac{20}{100} \cdot 1 \cdot 100 + (100{,}000 - x) \cdot \frac{10}{100} \cdot 1 \cdot 100 = 17{,}000 \cdot 100$

$\Rightarrow 20x + (100{,}000 - x) \cdot 10 = 1{,}700{,}000$

$\Rightarrow 20x + 1{,}000{,}000 - 10x = 1{,}700{,}000$

$\Rightarrow 10x = 700{,}000 \Rightarrow x = 70{,}000$

(b) $(\text{Total Interest}) = x \cdot \frac{20}{100} \cdot 1 + (100{,}000 - x) \cdot \frac{10}{100} \cdot 1 = 14{,}000$.

(multiply 100 on both-hand side!) $\Rightarrow x \cdot \frac{20}{100} \cdot 1 \cdot 100 + (100{,}000 - x) \cdot \frac{10}{100} \cdot 1 \cdot 100 = 14{,}000 \cdot 100$

$\Rightarrow 20x + (100{,}000 - x) \cdot 10 = 1{,}400{,}000$

$\Rightarrow 20x + 1{,}000{,}000 - 10x = 1{,}400{,}000$

$\Rightarrow 10x = 400{,}000 \Rightarrow x = 40{,}000$

Step 5 (a) $x = 70{,}000 \Rightarrow$ investing \$70,000 to stock A!

However, it does not satisfy the following condition:

client does not wish to invest more (to stock A) than \$50,000

Hence, there is no way to invest the money so that the annual rate is \$110,000

(b) $x = 40{,}000 \Rightarrow$ investing \$40,000 to stock A!

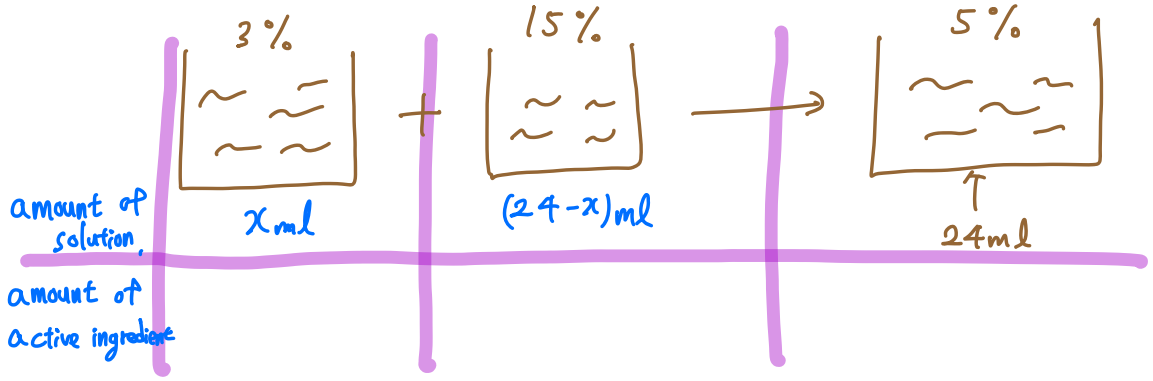
It satisfies the above condition, so investing \$40,000 to stock A, and \$(100,000 − 40,000) = \$60,000 to stock B is the way!

Ex (Preparing eye drops)

A pharmacist is to prepare 24 ml of special eye drops for a glaucoma patient. The eye-drop solution must have a 5 % ← Goal! active ingredient, but the pharmacist only have 15 % solution and 3 % solution in stock. How much of each type of solution should be used to fill the prescription.

* Mass percent formula

$$(\text{Mass percent of chemical}) = \frac{\text{mass of chemical}}{\text{Total mass of compound}} \times 100$$



Step 1 Let x (ml) be the amount of 3% solution that we use.

Then $(24-x)$ (ml) is the amount of 15% solution that we should use.

Step 2 We drew the pictures in the previous page.

Step 3 We want to construct an equation about the amount of active ingredient. (mass)

To do that we solved the formula

$$(\text{Mass percent of chemical}) = \frac{\text{mass of chemical}}{\text{Total mass of compound}} \times 100$$

for the mass of chemical!

We finished the class by getting

$$(\text{Mass of chemical}) = (\text{Total mass of compound}) \times \frac{(\text{Mass percent of chemical})}{100}$$

* We will continue tomorrow!