Section 2.1. Continued

* If the given equation consists of polynomials, then all the solutions that you get should be the solutions.

However, if the equation contains rational expressions or radical expressions you should check whether the solution yield a true statement when we replace x by the number in the original equation.

Ex $\frac{1}{x+2} + \frac{1}{x-1} = 3$ / $\sqrt{x-1} = 3$

If not, the solution is called extraneous solution (or root) and we do not consider it as a solution.

$x-1 = (x-1)'$
$x-3 = (x-3)'$
$2x-2 = 2'(x-1)'$
l.c.d = $2(x-1)(x-3)$

How to solve an equation containing rational expression.

1. Find l.c.d. of denominators.

$\frac{2}{x-1} + \frac{1}{x-3} = \frac{3}{2x-2}$

$x=1$ $x=3$ $x=1$

2. Find the values of x that make at least one denominator 0.
(These values cannot be solutions)

$2\cdot(x-1)\cdot(x-3)\left(\frac{2}{x-1} + \frac{1}{x-3}\right) = 2\cdot(x-1)\cdot(x-3)\frac{3}{2x-2}$

$2\cdot\cancel{(x-1)}\cdot(x-3)\cdot\frac{2}{\cancel{x-1}} + 2\cdot(x-1)\cancel{(x-3)}\cdot\frac{1}{\cancel{x-3}} = \cancel{2}\cdot\cancel{(x-1)}\cdot(x-3)\cdot\frac{3}{\cancel{2}(\cancel{x-1})}$

$4\cdot(x-3) + 2\cdot(x-1) = 3\cdot(x-1) \rightarrow 4x-12+2x-2 = 3x-9$

$6x-14 = 3x-9$
$-3x \quad -3x$

3. Multiply l.c.d on the both hand side and solve it.

$3x-14 = -9$
$+14 \quad +14$
$3x = 5$
$x = \frac{5}{3}$

4. From the solutions that you get, delete the ones from step 2.

$\frac{5}{3}$ is not the same as 1 or 3, so it is the solution.

Ex Solve: $\frac{4x}{x-3} = 2 + \frac{12}{x-3}$

Step 1. l.c.d $= (x-3)$

Step 2. Find the values of x that makes at least one of the denominators $= 0$. : $\boxed{x=3}$

Step 3. $\cancel{(x-3)}\cdot\frac{4x}{\cancel{x-3}} = (x-3)\cdot 2 + \cancel{(x-3)}\cdot\frac{12}{\cancel{x-3}}$

$4x = 2(x-3)+12$

$4x = 2x-6+12$

$4x = 2x+6$.
$-2x \quad -2x$
$2x = 6 \rightarrow \boxed{x=3}$.

Step 4. $\cancel{x=3}$: extraneous solution.

$\Rightarrow$ No solution !!!

Ex Solve : $\frac{-1}{4x-2} + \frac{3x-2}{x+3} = \frac{6x+1}{2x-1}$

Step 1

$4x-2 = 2(2x-1)$

$x+3 = (x+3)$

$2x-1 = (2x-1)$

l.c.d. $= 2(2x-1)(x+3)$

Step 2

$4x-2=0$ when $x=\frac{1}{2}$

$x+3=0$ when $x=-3$

$2x-1=0$ when $x=\frac{1}{2}$

$\rightarrow x=\frac{1}{2}$ and $x=-3$ cannot be the solutions.

Step 3

multiply l.c.d on the both hand side of the equation :

$$2(2x-1)(x+3)\cdot\frac{-1}{2(2x-1)} + 2(2x-1)(x+3)\cdot\frac{3x-2}{x+3} = 2(2x-1)(x+3)\cdot\frac{6x+1}{2x-1}$$

$$-(x+3) + 2(2x-1)(3x-2) = 2(x+3)(6x+1)$$

$$-(x+3) + 2(6x^2-7x+2) = 2(6x^2+19x+3)$$

$$-x-3+12x^2-14x+4 = 12x^2+38x+6$$

$$12x^2-15x+1 = 12x^2+38x+6$$

$$-15x+1 = 38x+6$$

$$-15x+1-38x = 38x+6-38$$

$$-53x+1 = 6$$

$$-53x+1-1 = 6-1$$

$$-53x = 5$$

$$x = \frac{5}{-53} = -\frac{5}{53}$$

$-\frac{5}{53}$ is different from $\frac{1}{2}$ and -3, so it is the solution!

When an equation involving several variables are given, we sometimes have to solve the equation for a variable.

* Consider other variables as if they are constants!

Ex Solve $a = \frac{b}{2-b}$ for b.

multiply l.c.d $\rightarrow$ $a\cdot(2-b) = (2-b)\cdot\frac{b}{2-b}$

$\rightarrow a(2-b) = b$

$\rightarrow 2a-ab = b$ (+ab, +ab)

$\rightarrow 2a = ab+b$

$2a = b(a+1)$

$\frac{2a}{a+1} = b$

Compare the proof!

$5 = \frac{b}{2-b}$ multiply l.c.d. $5\cdot(2-b) = (2-b)\cdot\frac{b}{2-b} \rightarrow 5\cdot2-5\cdot b = b$ (+5b, +5b)

$\rightarrow 5\cdot2 = (1+5)b$

$\rightarrow \frac{5\cdot2}{(1+5)} = b$

$b = \frac{2a}{a+1}$

$b =$ (some expression that only involves a, but not b.)

- Do the following Exercises by your own!

(Proof will be provided when I upload the lecture note)

① Solve $4x-11=7x+5$

$\rightarrow 4x-11-7x=7x+5-7x \Rightarrow -3x-11=5$

$\Rightarrow -3x-11+11=5+11$

$\Rightarrow -3x=16, \quad x=-\frac{16}{3}$

② Solve $(8x-3)(2x+4)=(4x-2)^2$

: Use FOIL $\rightarrow 16x^2+32x-6x-12=16x^2-16x+4$

$\rightarrow \cancel{16x^2}+26x-12=\cancel{16x^2}-16x+4$

$\rightarrow 26x-12=-16x+4$

$\rightarrow 26x-12+16x=-16x+4+16x$

$\rightarrow 42x-12=4$

$\rightarrow 42x-12+12=4+12$

$\rightarrow 42x=16$

$\rightarrow x=\frac{16}{42}=\frac{8}{21}$

③ Solve $\frac{4}{3x-6}-\frac{3}{x+4}=\frac{1}{x-2}$

Step 1 $3x-6=3\cdot(x-2)$

$x+4=\quad\ (x+4)$

$x-2=\quad\ (x-2)$

l.c.d $=3\cdot(x-2)\cdot(x+4)$.

Step 2 $3x-6=0$ when $x=2$

$x+4=0$ when $x=-4$

$x-2=0$ when $x=2$

$\Rightarrow x=2, x=-4$ cannot be the solutions

Step 3 multiply l.c.d. from the both hand side:

$\cancel{3\cdot(x-2)}(x+4)\cdot\frac{4}{\cancel{3(x-2)}}-3\cdot(x-2)\cancel{(x+4)}\cdot\frac{3}{\cancel{x+4}}=3\cdot\cancel{(x-2)}(x+4)\cdot\frac{1}{\cancel{x-2}}$

$4(x+4)-9\cdot(x-2)=3\cdot(x+4)$

$4x+16-9x+18=3x+12$

$-5x+34=3x+12$

$-5x+34-3x=\cancel{3x}+12-\cancel{3x}$

$-8x+34=12$

$-8x=12-34=-22 \Rightarrow x=\frac{-22}{-8}=\frac{11}{4}$

Since $\frac{11}{4}$ is different from 2 and -4, $x=\frac{11}{4}$ is the solution!

④ Solve $P=ab+2bc+3ca$ for c.

$6=1+2c+3c$

$6=1+5c$

$6-1=5c$

$1=\frac{5}{5}=\frac{6-1}{5}=c$

$P=ab+2bc+3ca$

$P=ab+(2b+3a)c$

$P-ab=(2b+3a)c$

$\frac{P-ab}{2b+3a}=c$

Section 2.2. Applied Problems (= Sentence Problems)

To solve such problems.

Step 1. You should set x = (unknown quantity that the problem is asking)

You can change it.

Step 2. Draw a picture if it is helpful.

Step 3. Using the given information, construct an equation.

Step 4. Solve the equation.

Step 5. Check whether the solution makes sense or not.

Ex (Test average)

* The average of n numbers $a_1, a_2, \cdots, a_n$ is $\dfrac{a_1+a_2+\cdots+a_n}{n}$.

I got 77, 95, 82 on my 1st, 3rd and 4th Biology exam. If the average is 85, what score did I get in the 2nd exam?

Proof) Let x be my score in the 2nd exam.

1st	77
2nd	x
3rd	95
4th	82

$\Rightarrow$ (average) $= \dfrac{77+x+95+82}{4} = 85$ (from the condition!)

multiply 4: $77+x+95+82 = 85\cdot 4$

$172 + x + 254 = 340$

$254 + x = 340$.

$x = 86$

$\Rightarrow$ I got 86 pt in the 2nd exam!

Ex Ian got 95, 65, and 75 on his first three French exam. The 4th exam consisted of 20 problems and each problem is worth 5 pt. If the averge of his four exams is 80, how many problem did Ian missed in his fourth exam?

Proof) Let x be the number of problems that Ian missed in his fourth exam.
Since each problem is worth 5 pt, he lost $5\cdot x$ point in the fourth exam.
Hence, he scored $(100-5x)$ pt in the fourth exam.

1st	95
2nd	65
3rd	75
4th	$100-5x$

$\Rightarrow$ (average) $= \dfrac{95+65+75+(100-5x)}{4} = 80$ (from the condition!)

multiply 4: $95+65+75+(100-5x) = 80\cdot 4$

$335 - 5x = 320$

$-5x = -15$,

$x = 3$.

$\Rightarrow$ Ian missed 3 problems in the fourth exam.