Section 1.4 Fractional Expressions

* MLC, ASC are available!
* My office hour : M 2:45-3:45
  W 11-12
* Please start the homework!

Fractional Expressions : $\frac{\text{(Algebraic Expressions)}}{\text{(Algebraic Expressions)}}$

Rational Expressions : $\frac{\text{(Polynomial)}}{\text{(Polynomial)}}$ Ex $\frac{x^3-3x+1}{x^2-3x+2}$

In this section, we will simplify rational expressions by using the following properties : ① $\frac{b}{a} \pm \frac{c}{a} = \frac{b \pm c}{a}$

② $\frac{1\cancel{b}c}{a\cancel{b}_1} = \frac{c}{a}$

* $\frac{(x+5)\cancel{(x-2)}}{(x-1)\cancel{(x-2)}} = \frac{x+5}{x-1}$

- Simplifying Product & Division of Rational Expression : ① Factor each polynomials.
② Cancel out common factors appeared on the top and the bottom.

Ex Simplify $\frac{x^2-x-2}{x^2-9} \div \frac{2x-4}{x^2+3x}$

$= \frac{x^2-x-2}{x^2-9} \times \frac{x^2+3x}{2x-4}$

$= \frac{\cancel{(x-2)}(x+1)}{\cancel{(x+3)}(x-3)} \times \frac{x\cancel{(x+3)}}{2\cancel{(x-2)}} = \frac{x(x+1)}{2(x-3)}$

- Simplifying Addition & Subtraction of Rational Expression

A polynomial is called irreducible or prime if it cannot be factored.

When we add or subtract rational expressions, we first have to find a "least common denominator" (l.c.d) of them.

How to find a l.c.d.?

Step 1) Factor all the denominators into irreducible polynomials.

Step 2) Gather all the irreducible polynomials and decide the exponents (finding the largest exponents!)

Ex Find l.c.d. of the denominators x^2-4x+4, x^2-4, and $x+2$.

$x^2-4x+4 = (x-2)^2$

$x^2-4 = (x-2)^1(x+2)^1$

$x+2 = (x+2)^1$

l.c.d $= (x-2)^2(x+2)^1$

Once we find l.c.d., rewrite each fractional expression into $\frac{(\text{polynomial})}{(\text{l.c.d.})}$ and perform addition or subtraction.

Ex Simplify $\frac{5}{12}+\frac{3}{28}=\frac{5\cdot 7}{2^2\cdot 3\cdot 7}+\frac{3\cdot 3}{2^2\cdot 7\cdot 3}=\frac{35}{84}+\frac{9}{84}$

$12=2^2\cdot 3^1$

$28=2^2\cdot 7^1$

$l.c.d=2^2\cdot 3^1\cdot 7^1=84$

$=\frac{35+9}{84}=\frac{44}{84}=\boxed{\frac{11}{21}}$

Ex Simplify $\frac{3x-2}{x^2-4x+4}+\frac{x-1}{x^2-4}-\frac{2}{x+2}$

$x^2-4x+4=(x-2)^2$

$x^2-4=(x-2)^1(x+2)^1$

$x+2=(x+2)^1$

$l.c.d=(x-2)^2(x+2)^1$

$=\frac{(3x-2)(x+2)}{(x-2)^2(x+2)}+\frac{(x-1)(x-2)}{(x+2)(x-2)(x-2)}-\frac{2(x-2)^2}{(x+2)(x-2)^2}$

$=\frac{(3x-2)(x+2)+(x-1)(x-2)-2(x-2)^2}{(x-2)^2(x+2)}$

$=\frac{3x^2+6x-2x-4+x^2-2x-x+2-2(x^2-4x+4)}{(x-2)^2(x+2)}$

$=\frac{3x^2+6x-2x-4+x^2-2x-x+2-2x^2+8x-8}{(x-2)^2(x+2)}=\boxed{\frac{2x^2+9x-10}{(x-2)^2(x+2)}}$

Complex fraction : $\frac{\text{(fractional expression)}}{\text{(fractional expression)}}$ / Use $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$

Ex) $\frac{\frac{x^2-3x+y}{x-y}}{\frac{2x-1}{4y}}$

Ex Simplify $\frac{\frac{3}{x+2}-\frac{3}{y+2}}{x-y}=\frac{\frac{3(y-x)}{(x+2)(y+2)}}{\frac{(x-y)}{1}}=\frac{3(y-x)}{(x+2)(y+2)(x-y)}=\frac{-3(x-y)}{(x+2)(y+2)(x-y)}=\boxed{\frac{-3}{(x+2)(y+2)}}$ answer!

$\frac{3(y-x)}{(x+2)(y+2)}$

Let's simplify the top : $\frac{3}{x+2}-\frac{3}{y+2}$

$(x+2)=(x+2)^1$

$(y+2)=(y+2)^1$

$l.c.d=(x+2)^1(y+2)^1$

$=\frac{3(y+2)}{(x+2)(y+2)}-\frac{3(x+2)}{(x+2)(y+2)}$

$=\frac{3(y+2)-3(x+2)}{(x+2)(y+2)}$

$=\frac{(3y+6)-(3x+6)}{(x+2)(y+2)}$

$=\frac{3y+6-3x-6}{(x+2)(y+2)}$

$=\frac{3y-3x}{(x+2)(y+2)}=\frac{3(y-x)}{(x+2)(y+2)}$: Not the answer!

Examples that I gave you 5 minutes in the class.

① Simplify $\dfrac{4x^2-1}{4x^2+4x-3} \cdot \dfrac{2x+3}{2x^2+x}$

$$= \frac{(2x+1)(2x-1)}{(2x+3)(2x-1)} \cdot \frac{(2x+3)}{x(2x+1)} = \frac{1}{x}.$$

② Simplify $\dfrac{4x}{3x+1} - \dfrac{2x+1}{3x^2+x} - \dfrac{3}{x} = \dfrac{4x \cdot x}{(3x+1)\cdot x} - \dfrac{2x+1}{3x^2+x} - \dfrac{3(3x+1)}{x\cdot(3x+1)}$

$3x+1 = (3x+1)'$

$3x^2+x = (3x+1)'x'$

$x = x'$

l.c.d $= (3x+1)'x'$

$$= \frac{4x^2-(2x+1)-3(3x+1)}{(3x+1)\cdot x}$$

$$= \frac{4x^2-2x-1-9x-3}{(3x+1)\cdot x}$$

$$= \frac{4x^2-11x-4}{(3x+1)\cdot x}$$