Section 1.3 continued

* Online Homework 1, Written Homework 1 have been posted!
due is Next Friday (9/3) at 1 pm.
* Textbook is available on Canvas
* If you want to review factoring, see my Algebra lecture note on the bottom of the course Canvas main page.

- (Recall) Useful formulas

① $(x+y)(x-y) = x^2 - y^2$

② $(x \pm y)^2 = x^2 \pm 2xy + y^2$

③ $(x \pm y)^3 = x^3 \pm 3x^2y + 3xy^2 \pm y^3$

④ $(\underset{a}{x} - y)(\underset{b}{x^2} + \underset{c}{xy} + \underset{d}{y^2}) = x^3 - y^3$ $= (x-y)x^2 + (x-y)xy + (x-y)y^2$
$= x^3 - \cancel{x^2y} + \cancel{x^2y} - \cancel{xy^2} + \cancel{xy^2} - y^3$

⑤ $(x+y)(x^2 - xy + y^2) = x^3 + y^3$

$(3a+2b)((3a)^2 - (3a)\cdot(2b) + (2b)^2) = (3a)^3 + (2b)^3 = 27a^3 + 8b^3$

Ex Multiply: $(3a+2b)(9a^2 - 6ab + 4b^2)$
$= (3a+2b)((3a)^2 - (3a)\cdot(2b) + (2b)^2) = \boxed{27a^3 + 8b^3}$

Conversely

Ex Factor: $27a^3 + 8b^3 = (3a)^3 + (2b)^3$
$= (3a+2b)(9a^2 - 6ab + 4b^2)$.

Greatest Common Factor (G.C.F) of monomials

is the $\boxed{\text{factor}}$ that has the ① largest coefficients and ② the most variables

<u>Ex</u> Find G.C.F of $4a^3bc^2$, $6a^2b^3c^4$, and $8a^4bc^3$

$$4a^3bc^2 = 2^2 \cdot 3^0 \cdot a^3\, b^1\, c^2$$
$$6a^2b^3c^4 = 2^1 \cdot 3^1 \cdot a^2\, b^3\, c^4$$
$$8a^4bc^3 = 2^3 \cdot 3^0 \cdot a^4 \cdot b^1 \cdot c^3$$

$$2^1 \cdot a^2 \cdot b^1 \cdot c^2 = \boxed{2a^2bc^2}$$

g.c.f. $2a^2bc^2$

<u>Ex</u> $\boxed{4a^3bc^2 - 6a^2b^3c^4 + 8a^4bc^3}$

$$= 2a^2bc^2 \cdot \underline{2a} - 2a^2bc^2 \cdot \underline{3b^2c^2} + 2a^2bc^2 \cdot \underline{4a^2c}$$
$$= 2a^2bc^2(2a - 3b^2c^2 + 4a^2c)$$

☆ Factoring Polynomials ☆

<u>Step 1</u> Factor out the g.c.f.

(or negative of g.c.f if the leading coefficient is negative)

↓

<u>Step 2</u> Try to ① use the formulas

② factor by <u>trial and error</u> (in case of <u>trinomial</u>)

③ factor by <u>grouping</u> (usually where there are <u>even monomials</u>)

↓

<u>Step 3</u> Reapeat Step 2 until we cannot factor further.

Ex ① Factor $32a^6b - 162a^2b^9$

$= 2^5a^6b - 2\cdot 3^4a^2b^9$; g.c.f $= 2a^2b$.

$x^2 - y^2 = (x+y)(x-y)$

$= 2a^2b\cdot 2^4a^4 - 2a^2b\cdot 3^4\cdot b^8$

$= 2a^2b(2^4a^4 - 3^4b^8)$

$= 2a^2b((2^2a^2)^2 - (3^2b^4)^2)$

$= 2a^2b(2^2a^2+3^2b^4)(2^2a^2-3^2b^4)$

$= 2a^2b(4a^2+9b^4)(4a^2-9b^4)$

$\to 2a^2b(4a^2+9b^4)((2a)^2-(3b^2)^2)$

$= \boxed{2a^2b(4a^2+9b^4)(2a+3b^2)(2a-3b^2)}$

② Factor $12y^2z + 12yz^2 - 45z^3$

$= 3z(4y^2 + 4yz - 15z^2)$

$= 3z(ay+bz)(cy+dz)$

$= acy^2 + (ad+bc)yz + bdz^2$

$\boxed{3z(2y+5z)(2y-3z)}$

$a\cdot c = 4$
$b\,d = -15$
$\boxed{ad+bc = 4}$ ✓

$(a,c) = (2,2)$ or $(1,4)$

$(b,d) = (1,-15)\ (-1,15)$
$(3,-5)\ (-3,5)$
$(5,-3)\ (-5,3)$
$(15,-1)\ (-15,1)$

$(a,c) = (2,2),\ (b,d) = (1,-15)$

$ad + bc = 2\cdot(-15) + 1\cdot 2$

$= -30 + 2 = -28 \neq 4.$

$\boxed{(a,c) = (2,2),\ (b,d) = (5,-3)}$

$ad + bc = 2\cdot(-3) + 5\cdot 2 = -6 + 10 = \boxed{4}$

③ Factor $-2x^2 - 3xy + 4x + 6y$

$-x(2x+3y) + 2(2x+3y)$

$\boxed{(-x+2)(2x+3y)}$

④ Factor $4x^2y - y^3 + 6y^2 - 9y$

$= y(4x^2 - y^2 + 6y - 9)$

$= y(4x^2 - (y^2 - 6y + 9))$

$y^2 - 2\cdot 3\cdot y + 3^2$

$= y(4x^2 - (y-3)^2)$

$(x-y)^2 = x^2 - 2xy + y^2$

$x^2 - y^2 = (x+y)(x-y)$

$$= y\left((2x)^2-(y-3)^2\right) = y(2x+(y-3))(2x-(y-3))$$
$$= \boxed{y(2x+y-3)(2x-y+3)}$$

- Do the following Exercises by your own!

(Proof will be provided when I upload the lecture note)

① Find the product : $(3x-2y)^3$

② Factor : $-15x^2+x+2$

③ Factor : $x^3-6x^2+12x-8$ (try to use one of the formulas!)

Pf) ① $(3x-2y)^2 = (3x)^2 - 2\cdot(3x)\cdot(2y) + (2y)^2 = \boxed{9x^2-12xy+4y^2}$

② $-15x^2+x+2 = -(15x^2-x-2) = -(ax+b)(cx+d)$
$$= -(acx^2+(ad+bc)+bd)$$

We need to find the numbers a, b, c, and d such that

1) $ac = 15$
2) $bd = -2$
3) $ad+bc = -1$

$\rightarrow$ $(a,c) = (3,5)$, $(b,d) = (1,-2)$ work!

$-15x^2+x+2 = \boxed{-(3x+1)(5x-2)}$

③ There can be two possible proofs.

First proof (using $(x-y)^3 = x^3-3x^2y+3xy^2-y^3$) $\cdots$ (*)

: $x^3-6x^2+12x-8 = x^3-3\cdot x^2\cdot 2+3\cdot x\cdot 2^2-2^3 = \boxed{(x-2)^3}$

Replace $y=2$ in (*)

Second proof (using grouping)

$$
\begin{aligned}
: x^3 - 6x^2 + 12x - 8 &= (x^3 - 8) - 6x^2 + 12x \\
&= (x^3 - 2^3) - 6x(x-2) \\
&= (x-2)(x^2+2x+4) - (x-2)\cdot 6x \\
&= (x-2)\{(x^2+2x+4) - 6x\} \\
&= (x-2)(x^2-4x+4) \\
&= (x-2)(x^2 - 2\cdot 2x + 2^2) \\
&= (x-2)(x-2)^2 \\
&= \boxed{(x-2)^3}
\end{aligned}
$$

* The proof of Section 1.3. Exercise #45 - #102 will be provided in a separate file.

Please try as many problems as you can, then compare your proofs & answers with mine.