Section 1.3 continued

* Please join WebAssign!
* Webwork 1 and Written HW 1 will be posted tonight. (due next Friday 1 pm)

(Recall)

Algebraic Expression

: An expression obtained by applying $+, -, \times, \div$, power, root to variables and constants.

(Domain of Algebraic Expression is postponed)

A monomial in x : ax^n, a is any real number, n is non-negative integer. Ex $2x^3$, ~~$-2x^{-2}$~~

A binomial in x : Sum of two monomials. Ex $3x^6 - 2x = 3x^6 + (-2x)$

A trinomial in x : Sum of three monomials Ex $3x^3 - 3x + 7 = 3x^3 + (-3x) + 7x^0$

A polynomial in x : Sum of finitely many monomials

$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ with $a_n \neq 0$.

degree of the polynomial. n

leading coefficient. a_n

Ex $3x^2 - 4x + 1$

$-2x + 5x^4 - 3x^2 + 3 = 5x^4 - 3x^2 - 2x + 3$

we wrote the polynomial in decreasing order of exponents

* General form of the Distributive Property

: $a(b+c) = ab + ac$ / $(a+b)c = ac + bc$ $\quad 3x + 2x = (3+2)\cdot x = 5x$

$a(b+c+d) = ab+ac+ad$ / $(a+b+c)d = ad+bd+cd$

$a(b+c+d+e) = ab+ac+ad+ae$

Ex ① $3x$ and $2x$ ③ ~~$2x^2y$ and $3xy^2$~~

② ~~$3x$ and $2y$~~ $\quad 3x+2x =$

Two monomials are called like terms if they have the same variable part.

- Addition / Subtraction of polynomials : we can only combine like terms!

Find the difference

Ex $(3x^3 - 4x^2 - 3x + 1) - 2(x^3 - 2x^2 + 2x - 3)$

$3x^3 - 4x^2 - 3x + 1 - 2x^3 + 4x^2 - 4x + 6$

$= 3x^3 - 2x^3 - 4x^2 + 4x^2 - 3x - 4x + 1 + 6 = x^3 - 7x + 7$

- Multiplication of polynomials : use Distributive Property!

(or FOIL when you multiply binomials)

Find the product

Ex $\underbrace{(2x-3)}_{a}\underbrace{(3x+5)}_{b\ \ c} = \overbrace{(2x-3)}^{2x+(-3)}\cdot 3x + (2x-3)\cdot 5$

$a(b+c) = ab+ac$

$(a+b)c = ac+bc$

$= 6x^2 - 9x + 10x - 15$

$= 6x^2 + x - 15$

FOIL

$(a+b)(c+d)$

$= ac + ad + bc + bd$

- Dividing a polynomial by a monomial

: Rewrite $\frac{(\text{Polynomial})}{(\text{monomial})} \Rightarrow$ sum of $\frac{(\text{monomial})}{(\text{monomial})}$ $\quad \frac{a+b+d}{c} = \frac{a}{c} + \frac{b}{c} + \frac{d}{c}$

Ex Express as a polynomial in a & b : $\frac{9a^5b^3 - 3a^2b^4 + 6a^3b^6}{3a^2b^3}$

$= \frac{3\cancel{9}a^5b^3}{1\cancel{3}a^2b^3} - \frac{\cancel{3}a^2b^4}{\cancel{3}a^2b^3} + \frac{2\cancel{6}a^3b^6}{1\cancel{3}a^2b^3}$ (a^{3-2}, b^{6-3})

$= 3a^3 - b + 2ab^3$

Ex ① Find the sum : $3(2x^4 - 3x^3 + x^2 - 3) + 2(3x^4 + 4x^3 - 2x + 5)$

$= 6x^4 - 9x^3 + 3x^2 - 9 + 6x^4 + 8x^3 - 4x + 10$

$= 12x^4 - x^3 + 3x^2 - 4x + 1$

② Find the product : $\underset{a}{(y+2)}\underset{b}{(2x^2}\underset{c}{-3xy}\underset{d}{+x)}$

$= (y+2)\cdot 2x^2 + (y+2)\cdot(-3xy) + (y+2)\cdot x$

$= 2x^2y + 4x^2 - 3xy^2 - 6xy + xy + 2x$

$= 2x^2y + 4x^2 - 3xy^2 - 5xy + 2x$

- Useful formulas

① $(x+y)(x-y) = x^2 - y^2$

② $(x \pm y)^2 = x^2 \pm 2xy + y^2$ → $(x+y)^2 = x^2 + 2xy + y^2$ and $(x-y)^2 = x^2 - 2xy + y^2$.

③ $(x \pm y)^3 = x^3 \pm 3x^2y + 3xy^2 \pm y^3$

④ $(x-y)(x^2+xy+y^2) = x^3 - y^3$

⑤ $(x+y)(x^2-xy+y^2) = x^3 + y^3$

★ Factoring Polynomials ★

Step 1 Factor out the g.c.f. (or negative of g.c.f if the leading coefficient is negative)

ex) $3x^2y + 6xy^2 + 3y^3$

$3y \cdot x^2 + 3y \cdot 2xy + 3y \cdot y^2 = 3y(x^2 + 2xy + y^2)$

↓

Step 2 Try to ① use the formulas

② factor by trial and error (in case of trinomial)

③ factor by grouping (usually where there are even monomials)

↓

Step 3 Reapeat Step2 until we cannot factor further.

Ex ① Factor $16a^5b - 54a^2b^7$

$16 = 2^4$

$54 = 2^1 \cdot 3^3$

2^1

g.c.f = $2^1a^2b^1$