My office hour : M 2:45-3:45 / W 11-12
MLC is available from next Monday (Swain East 340).

Section 1.2 continued

$\sqrt[n]{a}$: n-th root of a.

When n is odd, $\sqrt[n]{a^n} = a$
When n is even, $\sqrt[n]{a^n} = |a|$.) ... ①

From now on, assume all letters are positive real numbers.

By ①, when a is positive, we have $\sqrt[n]{a^n} = a$. $\sqrt[3]{2^3} = 2$, $\sqrt[4]{2^4} = 2$.

We can use this result to remove the n-th power from the radicant:

Ex ① $\sqrt{8a^3b} = \sqrt{2^3 \cdot a^3 \cdot b} = \sqrt{2^2 \cdot 2 \cdot a^2 \cdot a \cdot b} = \sqrt{2^2 a^2 \cdot 2ab} = \sqrt{(2a)^2 \cdot 2ab} = \sqrt{(2a)^2} \cdot \sqrt{2ab}$
$= 2a\sqrt{2ab}$

② $\sqrt[4]{a^6 b^5} = \sqrt[4]{a^4 b^4 a^2 b} = \boxed{ab\sqrt[4]{a^2 b}}$.

③ $\sqrt[3]{27a^{10}b^5c^7} = \sqrt[3]{3^3 a^{10} b^5 c^7} = \sqrt[3]{3^3 \cdot (a^3)^3 \cdot b^3 (c^2)^3 \cdot ab^2c}$
$a^{10} = a^9 \cdot a = (a^3)^3 \cdot a$, $b^5 = b^3 \cdot b^2$, $c^7 = c^6 \cdot c = (c^2)^3 \cdot c$
$= \sqrt[3]{3^3} \cdot \sqrt[3]{(a^3)^3} \cdot \sqrt[3]{b^3} \cdot \sqrt[3]{(c^2)^3} \cdot \sqrt[3]{ab^2c}$
$= \boxed{3 \cdot a^3 b c^2 \sqrt[3]{ab^2c}}$

Suppose we have a rational expression whose denominator is of the following form: $\sqrt[n]{a^k}$ where $1 \le k < n$, $a > 0$.

(Ex $\dfrac{1}{\sqrt[5]{a^2}} \times \dfrac{\sqrt[5]{a^3}}{\sqrt[5]{a^3}} = \dfrac{\sqrt[5]{a^3}}{\sqrt[5]{a^5}} = \dfrac{\sqrt[5]{a^3}}{a}$)

We would like to rationalize a denominator!

multiply $\sqrt[n]{a^{n-k}}$ on both the top and the bottom so that the bottom becomes a.

$\sqrt{5} = \sqrt[2]{5^1}$ $\quad \sqrt[n]{a^k}$ multiply $\sqrt[n]{a^{n-k}}$

Ex ① $\sqrt{\frac{4}{5}} = \frac{\sqrt{4}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\cdot\sqrt{5}}{(\sqrt{5})^2} = \boxed{\frac{2\sqrt{5}}{5}}$

② $\sqrt[4]{\frac{b^2}{a^7}} = \frac{\sqrt[4]{b^2}}{\sqrt[4]{a^7}} = \frac{\sqrt[4]{b^2}}{\sqrt[4]{a^4 a^3}} = \frac{\sqrt[4]{b^2}}{a\cdot\sqrt[4]{a^3}} \cdot \frac{\sqrt[4]{a}}{\sqrt[4]{a}} = \frac{\sqrt[4]{b^2}\cdot\sqrt[4]{a}}{a\cdot\underbrace{\sqrt[4]{a^4}}_{=a}} = \boxed{\frac{\sqrt[4]{ab^2}}{a^2}}$

$\sqrt[n]{a^k}\cdot\sqrt[n]{a^{n-k}}$

- Rational Exponents

: Let $\frac{m}{n}$ be a rational number such that n is a positive integer greater than n.

$a^{\frac{m}{n}}$ when $m=1$

If $\sqrt[n]{a}$ exists, then 1) $a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = a^{\frac{1}{n}\cdot m} = (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m$

2) $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

$a^{\frac{m}{n}} = a^{m\cdot\frac{1}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$

3) $a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (a^m)^{\frac{1}{n}}$

* Law of exponents still holds when m and n are any real numbers.

① $a^m a^n = a^{m+n}$

$a^{\sqrt{2}} a^{\pi} = a^{\sqrt{2}+\pi}$

② $(a^m)^n = a^{mn}$

③ $(ab)^n = a^n b^n$

④ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

⑤ $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$. $\quad a^{-n} = \frac{1}{a^n}$

⑥ $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$

⑦ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Ex ① $(-8)^{\frac{4}{3}}\cdot 9^{-\frac{1}{2}} = (\sqrt[3]{-8})^4\cdot\frac{1}{9^{\frac{1}{2}}} = (-2)^4\cdot\frac{1}{3} = 16\cdot\frac{1}{3} = \boxed{\frac{16}{3}}$

② $(a^3 b^4)^{\frac{1}{2}} = (a^3)^{\frac{1}{2}}\cdot(b^4)^{\frac{1}{2}} = a^{\frac{3}{2}}\cdot b^2 = \sqrt{a^3}\cdot b^2 = a\cdot\sqrt{a}\cdot b^2 = \boxed{ab^2\sqrt{a}}$

③ $\left(\frac{3a^{\frac{1}{4}}}{b^{\frac{1}{3}}}\right)^2\left(\frac{b^{\frac{5}{3}}}{a^{\frac{1}{2}}}\right) = \frac{(3a^{\frac{1}{4}})^2}{(b^{\frac{1}{3}})^2}\left(\frac{b^{\frac{5}{3}}}{a^{\frac{1}{2}}}\right) = \frac{9\cdot\cancel{a^{\frac{1}{2}}}}{b^{\frac{2}{3}}}\cdot\frac{b^{\frac{5}{3}}}{\cancel{a^{\frac{1}{2}}}} = \frac{9\cdot b^{\frac{5}{3}}}{b^{\frac{2}{3}}} = \frac{9\cdot b^{\frac{5}{3}-\frac{2}{3}}}{1} = \frac{9b}{1} = \boxed{9b}$

$$\sqrt[4]{a^2} \cdot \sqrt[4]{a} = \sqrt[4]{a^3}$$

Using the law of exponents, we can combine radicals with different indices.

$\frac{1}{4}+\frac{1}{3}=\frac{3}{12}+\frac{4}{12}=\frac{7}{12}$

<u>Ex</u> Simplify

① $\sqrt[4]{a}\,\sqrt[3]{a} = a^{\frac{1}{4}} \cdot a^{\frac{1}{3}} \overset{①}{=} a^{\frac{1}{4}+\frac{1}{3}} = a^{\frac{7}{12}} = \boxed{\sqrt[12]{a^7}}$

② $\dfrac{\sqrt[3]{b}}{\sqrt[4]{b^3}} = \dfrac{b^{\frac{1}{3}}}{b^{\frac{3}{4}}} \underset{⑤}{=} \dfrac{1}{b^{\frac{3}{4}-\frac{1}{3}}} = \dfrac{1}{b^{\frac{5}{12}}} = \boxed{\dfrac{1}{\sqrt[12]{b^5}}}$

Section 1.3. Algebraic Expressions.

(Domain...)

Algebraic Expression

: An expression obtained by applying $+, -, \times, \div$, power, root to variables and constants.

$x, y, a, b \cdots$

$1, -3, \pi, \sqrt{2}$

<u>Ex</u> $\dfrac{2x^2 - \sqrt[3]{x-3}}{x^2+1}$