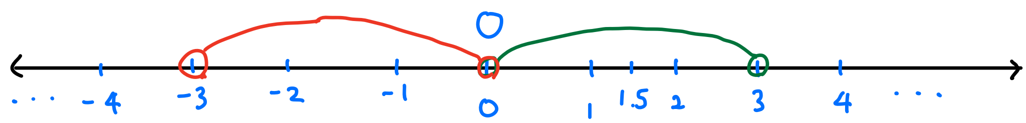
Section 1.1 continued

* Late HW is not accepted. Instead, a few of your lowest HW grades will be ignored.
* Please make WebAssign account and enter the class key!

Every number can be presented on a number line.



The absolute value of a real number a is a distance between a and 0 on the number line.

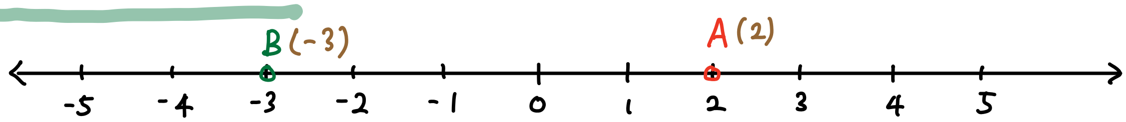
Ex $|3| = 3$, $|-3| = 3$ $\Rightarrow$ In general, $|a| = |-a|$ for any real number a.

$|2| = 2$, $|-5| = 5$

$$\Rightarrow \boxed{|a| = \begin{cases} a & \text{if } a \text{ is positive or } 0 \\ -a & \text{if } a \text{ is negative.} \end{cases}}$$

The distance between two points A and B whose coordinates are a and b is denoted by $d(A,B)$, and defined by

$d(A,B) = |b-a|$. $\qquad d(A,B) = |-3-2| = |-5| = 5$



$x < 2$

$\downarrow$

$x-2 < 2-2$

$\downarrow$

$x-2 < 0 \rightarrow x-2$ is negative.

Ex① When $x < 2$, simplify $|x-2| = -(x-2) = -x+2$

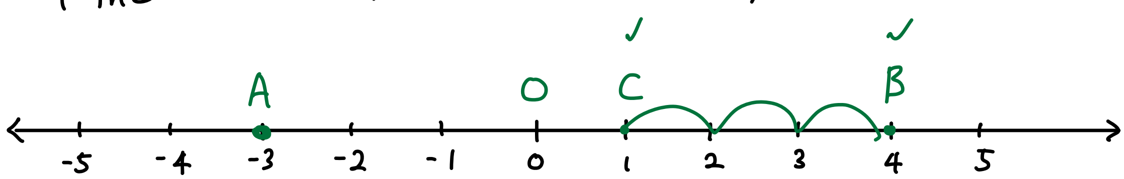
② When x is a real number, simplify $|x^2+2| = x^2+2$

Q. Is x^2+2 positive or negative?

x is a real number $\Rightarrow x^2 \geq 0 \Rightarrow x^2+2 \geq 2 > 0$: x^2+2 is positive!

Ex Let A, B and C have coordinates -3, 4, and 1.

Find $d(A,B)$, $d(B,C)$, $d(B,O)$, and $d(C,C)$



$d(A,B) = |4-(-3)| = |4+3| = 7.$

$d(B,C) = |1-4| = |-3| = 3$

$d(B,O) = 4$

$d(C,C) = 0.$

Section 1.2. Exponents and Radicals.

exponent

a^n : a to the n th power

base

When n is positive integer : $a^n = \overbrace{a \cdot a \cdot \cdots \cdot a}^{n \text{ copies}}$

n is zero : $\underline{a^0} = 1$ ex) $(1301)^0 = 1$

n is negative integer : $\underline{a^n = \frac{1}{a^{-n}}}$ ex) $a^{-3} = \frac{1}{a^3}$

PEMDAS.

Ex ① $(-\sqrt{5})^0 = 1$

② $(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{-8} = -\frac{1}{8}$.

③ $\frac{1}{4}\boxed{(-2)^3} - 16 \cdot \boxed{2^{-4}} = \frac{1}{4} \cdot (-8) - 16 \cdot \frac{1}{16} = -2 - 1 = -3$.

* Law of exponents.

For any real number a and b and any integers m and n,

① $a^m a^n = a^{m+n}$

② $(a^m)^n = a^{mn}$

③ $(ab)^n = a^n b^n$

④ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

⑤ $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$

⑥ $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$

⑦ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

"Simplifying" means a) each variable appears only once,

b) all exponents are positive.

<u>Ex</u> Simplify the following expressions: $\quad a^n = \dfrac{1}{a^{-n}}$

① $\left(\dfrac{a^{-2}}{2b^3}\right)^2 = \dfrac{(a^{-2})^2}{(2b^3)^2} = \dfrac{a^{-4}}{2^2\cdot(b^3)^2} = \dfrac{a^{-4}}{4b^6} = \boxed{\dfrac{1}{4a^4b^6}}$

② $\left(\dfrac{3a^2b}{c}\right)^3\left(\dfrac{b}{3a^3}\right)^2 = \dfrac{(3a^2b)^3}{c^3}\cdot\dfrac{b^2}{(3a^3)^2}$

$= \dfrac{\overset{3}{\cancel{27}}\,\overset{1}{\cancel{a^6}}\,b^3}{c^3}\cdot\dfrac{b^2}{\underset{1}{\cancel{9}}\,\underset{1}{\cancel{a^6}}}$

$= \boxed{\dfrac{3\cdot b^5}{c^3}}$

index

$\sqrt[n]{a}$: radical

radical symbol

radicand

n-th root of a.

For any positive integer n and a real number a, $\sqrt[n]{a}$ is defined as follows:

principal n-th root

a) If $a=0$, $\sqrt[n]{a}=\sqrt[n]{0}=0$

b) If $a>0$, $\sqrt[n]{a}$ is a positive real number whose n-th power is a.

c) 1) If $a<0$ and n is odd, $\sqrt[n]{a}$ is a negative real number, whose n-th power is a.

2) If $a<0$ and n is even, $\sqrt[n]{a}$ is not a real number.

Ex ① $\sqrt[3]{0} = 0$

② $\sqrt[4]{16}$ = a positive number whose 4-th power is 16. $= 2$.

③ $\sqrt[3]{-27}$ = a negative number whose cube is $-27 = -3$.

④ $\sqrt[4]{-16}$ = NOT a real number.

When $n=2$, we use $\sqrt{a}$ instead of $\sqrt[2]{a}$.

When $\sqrt[n]{a}$ is a real number, ① $(\sqrt[n]{a})^n = a$

and ② it has the same sign as a.

Properties of Radicals:

① $\sqrt[n]{a^n} = \begin{cases} a & \text{if } n \text{ is odd,} \\ |a| & \text{if } n \text{ is even.} \end{cases}$

$\sqrt[3]{(-3)^3} = \sqrt[3]{-27} = -3.$

$\sqrt[4]{(-3)^4} = \sqrt[4]{81} = 3$

② $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}.$

③ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}.$

④ $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}.$