Chapter 1. Fundamental concepts of Algebra

Section 1.1. Real Numbers.

Natural Numbers (= Positive integers) = $\{1, 2, 3, 4, \cdots\}$

Whole Numbers = $\{0, 1, 2, 3, 4, \cdots\}$

Integers = $\{\cdots -3, -2, -1, 0, 1, 2, 3, \cdots\}$

are numbers that can only be divided by 1 and itself.

Natural Numbers
- Prime Numbers = $\{2, 3, 5, 7, 11, \cdots\}$
- Composite Numbers = $\{4, 6, 8, 9, 10, 12, 14, \cdots\}$
- 1

it can be divided by 5.

Ex 34 is a composite number because $2 \times 17 = 34$.

* Every positive integer greater than 1 can be uniquely expressed as a product of prime numbers.

Ex $60 = 2 \cdot 30 = 2 \cdot 5 \cdot 6 = 2 \cdot 5 \cdot 2 \cdot 3 = 2^2 \cdot 3 \cdot 5$.

$24 = 2 \cdot 12 = 2 \cdot 2 \cdot 6 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3$.

Rational Number   Ex $\frac{3}{2}$, $-\frac{11}{4}=\frac{-11}{4}$, $5=\frac{5}{1}$

is a real number that can be expressed as a ratio of two integers,

Irrational Numbers   Ex π, $\sqrt{3}$, $-\sqrt{2}$, ...

is a real number that cannot be expressed as a ratio of two integers,

Real Number = Rational Number + Irrational Numbers

Equality / Inequality Symbols

Symbol	Read as	Example
$a=b$	a is equal to b	$3=3$
$a\neq b$	a is not equal to b	$2\neq 4$
$a<b$	a is less than b / b is greater than a	$2<4$, $-3<7$
$a>b$	a is greater than b / b is less than a	$4>2$, $7>-3$

$\leq$: < + =

$a \leq b$ — a is less than or equal to b; b is greater than or equal to a. $3 \leq 3$ $2 \leq 7$

$a \geq b$ — a is greater than or equal to b; b is less than or equal to a. $4 \geq 4$, $5 \geq 1$

$a \nless b$ — a is not less than b / b is not greater than a. $5 \nless 2$

⋮

* $a < b < c$ means $a < b$ and $b < c$.

In this case, we say b lies between a and c.

Ex $-1 < x < 3$: x lies between -1 and 3.

Properties of addition / multiplication / subtraction / division.

1) When we add / multiply several numbers, order does not matter.

Ex $2+5 = 5+2$ $\quad$ $2 \cdot 5 = 5 \cdot 2$

$(2+3)+5 = 2+(3+5)$ $\quad$ $(2 \cdot 3) \cdot 5 = 2 \cdot (3 \cdot 5)$

2) 0 is called additive identity / 1 is called multiplicative identity.

$3+0=3$ $\quad$ $5 \cdot 1 = 5$

$-4 \to -(-4) = 4$.

$4 \to -4$

3) For any real number a, $-a$ is called additive inverse or negative of a.

$4+(-4)=0$.

$(-4)+4=0$.

For any non-zero real number a, $\frac{1}{a} = a^{-1}$ is called (multiplicative inverse or reciprocal) of a.

$5 \longrightarrow \frac{1}{5} = 5^{-1}$

$5 \cdot \frac{1}{5} = 1$ $\quad$ $\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$

4) Distributive property : For any real numbers a, b, and c,

$$a(b+c) = ab + ac$$

$$(a+b)c = ac + bc$$

Ex $\underbrace{(x+2)}_{a}(\underbrace{x}_{b}+\underbrace{3}_{c}) = (x+2)\cdot x + (x+2)\cdot 3 = x^2 + 2x + 3x + 6 = \underline{x^2 + 5x + 6}$

$\underline{(5)}\cdot 0 = 0$

5) If we multiply 0 to any real number, the result is 0.

6) If a product of two numbers is 0, then at least one of them is 0.

$a \cdot b = 0 \Rightarrow a = 0$ or $b = 0$.

Ex $\underline{(x-2)}\underline{(x+3)} = 0.$

$x-2=0$ or $x+3=0 \Rightarrow x=2$ or $x=-3$

Subtracting b = adding $(-b)$, $\quad 3-5 = 3+(-5)$

Dividing b = multiplying $\frac{1}{b}$ or b^{-1} $\quad 3 \div 5 = 3 \cdot \frac{1}{5}$

* Order does matter when we do subtraction or division.

Ex. $2-5 \neq 5-2$ $\quad 2 \div 5 \neq 5 \div 2$

$2+(-5) \neq (-5)+2.$ $\quad 2 \cdot \frac{1}{5} \neq \frac{1}{5} \cdot 2.$

* $\underline{-a = (-1)\cdot a}$

$3 \cdot 4 = 12 \qquad (-3)\cdot(-4) = (-1)\cdot 3 \cdot (-1) \cdot 4 = 12.$

$\Rightarrow$ If two numbers have the same sign, their product is positive.

$\overset{N}{(-3)} \cdot \overset{P}{4} = (-1)\cdot 3 \cdot 4 = (-1)\cdot 12 = \overset{N}{-12}$

$\Rightarrow$ If two numbers have opposite sign, their product is negative.

$a = b \rightarrow$ $a+c=b+c$, $a-c=b-c$, $a\cdot c = b\cdot c$, $\frac{a}{c} = \frac{b}{c}$ if $c \neq 0$.

7) Adding / Subtracting / Multiplying the same real number preserves the equality.
Dividing the same non-zero real number preserves the equality.

8) Adding / Subtracting the same real number preserves the inequality.
Multiplying / Dividing the same positive real number preserves the inequality.
Multiplying / Dividing the same negative real number flips the inequality.

$a > b \longrightarrow a\cdot c < b \cdot c$

c is a negative

When you multiply or divide quotients, try to simplify / cancel out as soon as you can.

Ex $\frac{\cancel{6}\,\cancel{2}\,1}{\cancel{9}\,3} \times \frac{\cancel{14}\,7}{\cancel{20}\,\cancel{10}\,5} = \frac{7}{15}$

Ex $\frac{6}{8} \div \frac{30}{24} = \frac{\cancel{6}\,3}{\cancel{8}\,\cancel{4}\,1} \cdot \frac{\cancel{24}\,4}{\cancel{30}\,5} = \frac{3}{5}$.